

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

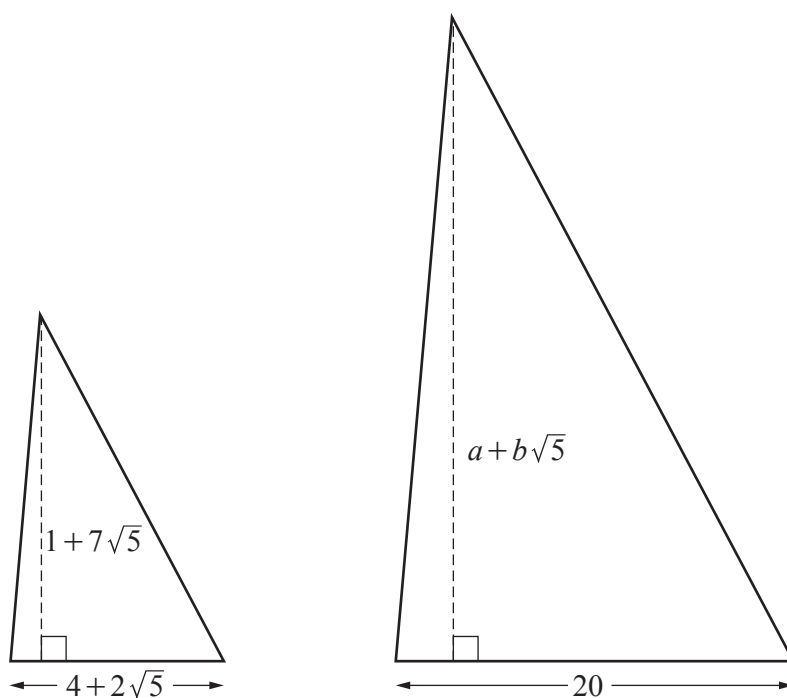
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Solve the equation $2|8-4x|+5=25$. [3]

- (b) Solve the inequality $16x-5x^2-3 < \frac{57-9x}{6}$. [4]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question all lengths are in centimetres.



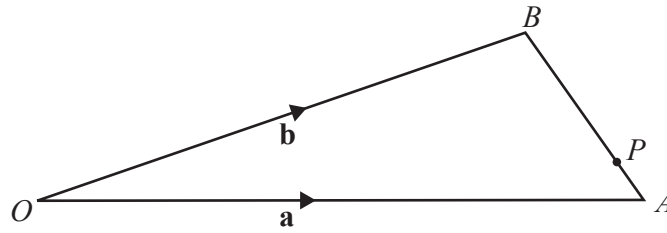
The diagram shows two similar triangles.

The height of the smaller triangle is $1 + 7\sqrt{5}$ and the height of the larger triangle is $a + b\sqrt{5}$, where a and b are integers.

Find the values of a and b .

[4]

3 (a)



The diagram shows a triangle OAB . The point P lies on AB . The ratio $AP:PB$ is $1:3$.

Given that $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$, find an expression for $\vec{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Simplify your answer. [2]

(b) Vector $\mathbf{q}$ has magnitude $12\sqrt{5}$ and direction $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

Vector $\mathbf{r}$ has magnitude $15\sqrt{2}$ and direction $\begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

Find the unit vector in the direction of $\mathbf{q} + \mathbf{r}$.

[6]

- 4 (a) (i) Given that $y = 3 \sin^2 x + \cos x$, show that $y + \cot x \frac{dy}{dx} = k(1 + \cos^2 x)$, where k is an integer. [4]

- (ii) Using your value of k , solve the equation $k(1 + \cos^2 x) = 4$ for $-\pi \leq x \leq \pi$. [4]

(b) (i) Differentiate $y = \tan(x - \sqrt{x})$ with respect to x . [2]

(ii) Hence find $\int \frac{2\sqrt{x} - 1}{\sqrt{x} \cos^2(x - \sqrt{x})} dx$. [2]

5 Variables x and y are related by the equation $y = \frac{x}{\ln 3x}$. Use differentiation to find the approximate change in y when x increases from 1 to $1 + h$, where h is small. [4]

- 6 Find the exact area of the region enclosed by the curve $y = e^{2-4x}$, the x -axis, the line $x = -0.25$ and the line $x = 0.5$. [4]

- 7 (a) The curves $4x^2 - 3y^2 + xy = 24$ and $y = \frac{2}{x}$ intersect at the points P and Q . Find the coordinates of P and Q . [5]

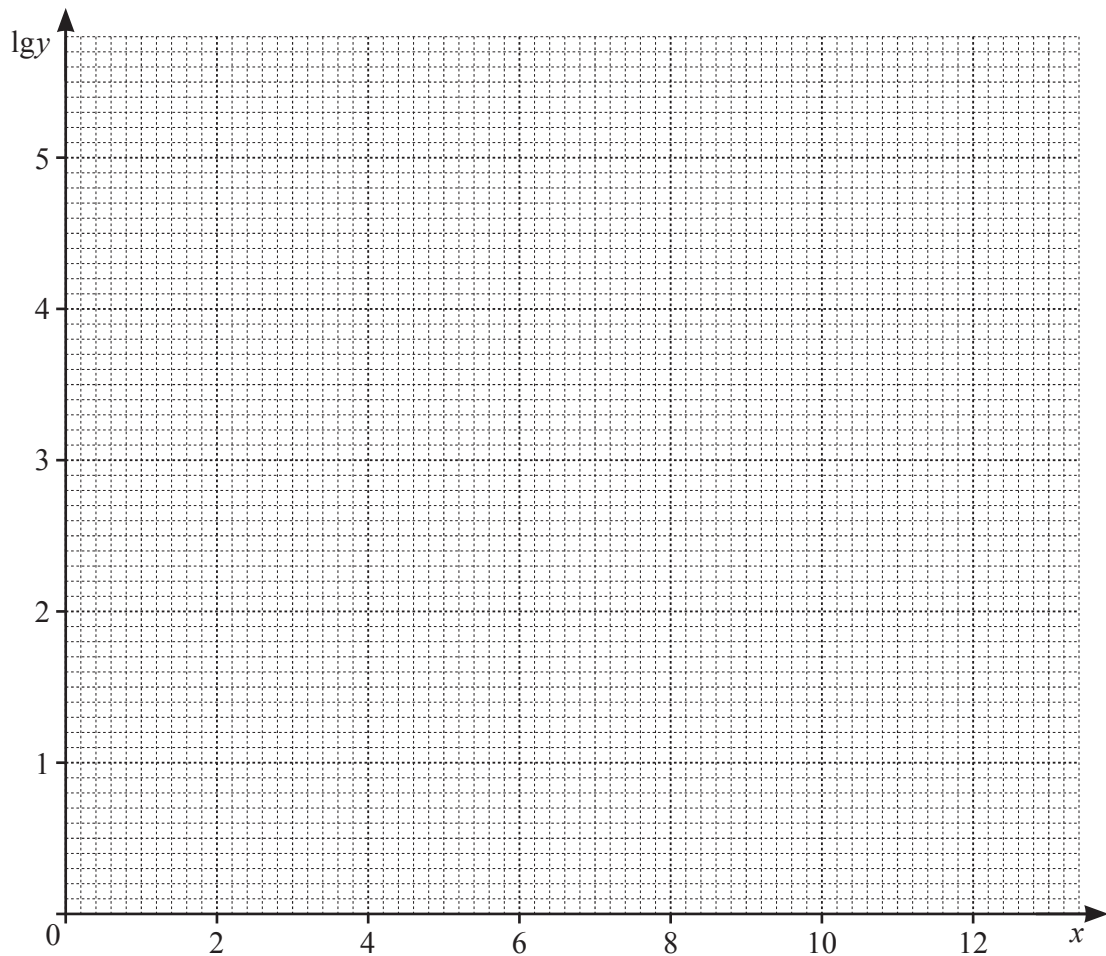
- (b) Find the length of PQ . Give your answer in the form $a\sqrt{b}$, where a is rational and b is the smallest possible integer. [2]

- 8 Variables y and x are known to be connected by the relationship $y = Ab^x$ where A and b are constants. The table shows values of y for certain values of x .

x	1	3	5	10	12
y	38	150	600	20 500	82 000

- (a) Draw the graph of $\lg y$ against x .

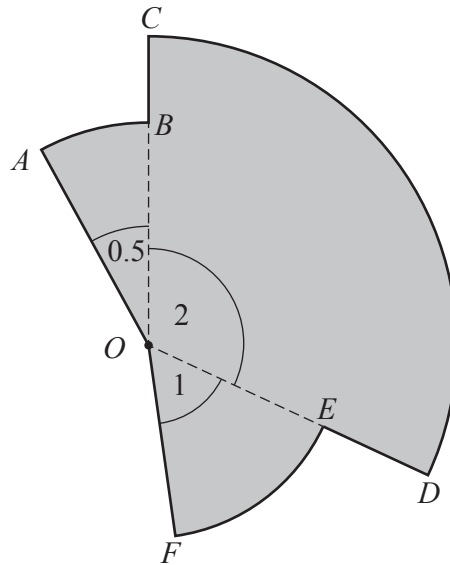
[2]



(b) Use your graph to find values of A and b , giving each to 1 significant figure. [6]

(c) Find an estimate of x when $y = 1500$. [2]

- 9 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a company logo. Each part of the logo is a sector of a circle with centre O .

Sector AOB has radius x .

Sector COD has radius $x + 2$.

Sector EOF has radius y .

The shaded region has area $A \text{ cm}^2$ and perimeter 24.

It is given that x and y can vary.

- (a) Show that $A = \frac{91}{8}x^2 - 68x + 132$.

[4]

(b) Use differentiation to find the minimum possible area of the logo.

[5]

- 10** The expansion of $\left(a + \frac{x}{a}\right)^n$ in ascending powers of x begins $b^4 + 48b^3x$, where n , a and b are positive integers.

(a) Show that $a^{\frac{n}{2}-4} = \left(\frac{48}{n}\right)^2$. [4]

- (b) Given also that the third term is $1056b^2x^2$, find the values of n , a and b . [6]

Question 11 is printed on the next page.

- 11 A cylinder, open at both ends, has base radius r cm and height $4r$ cm. Its curved surface area is S cm².

Given that r varies with time t , find S at the instant when $\frac{dS}{dt} = 6 \frac{dr}{dt}$. [5]

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