

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

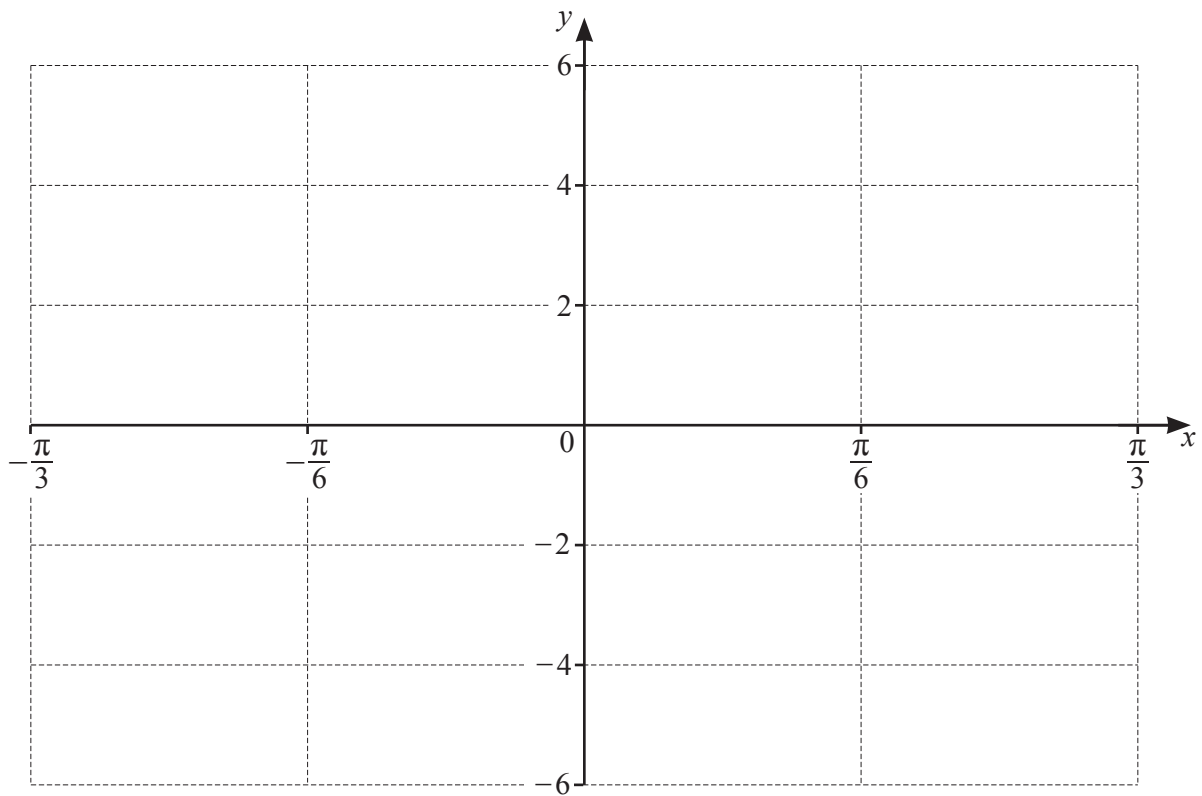
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 On the axes, sketch the graph of $y = 4 \sin 3x - 2$ for $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

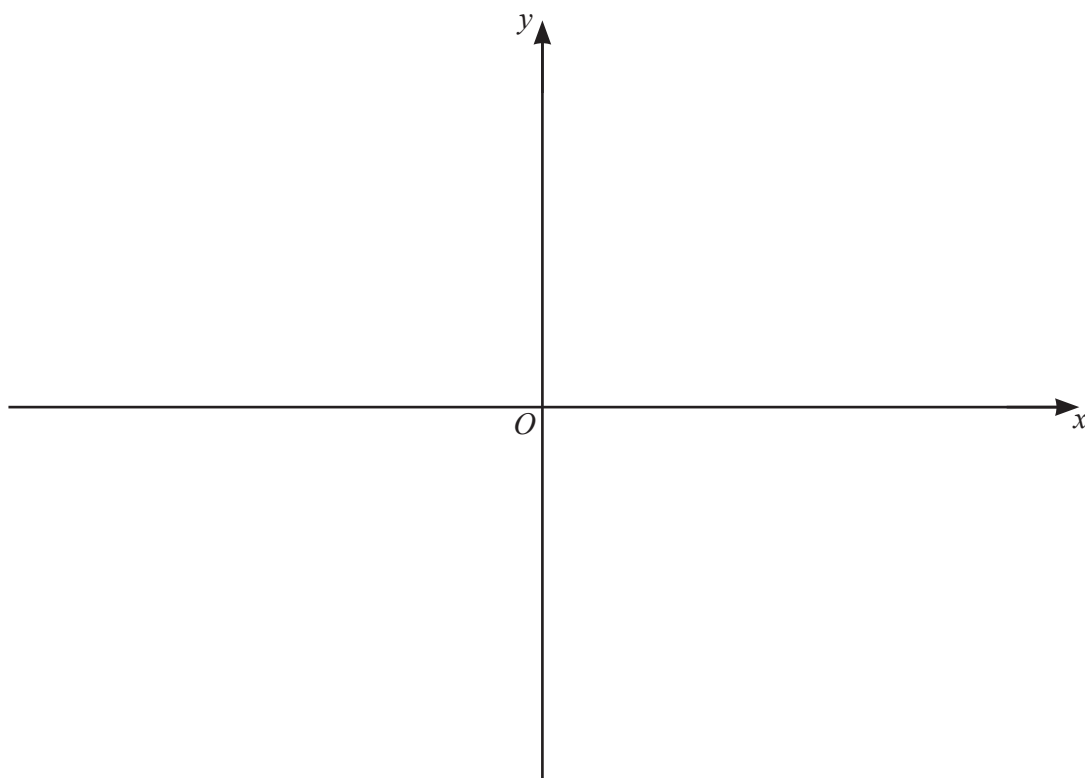
[3]



- 2 (a) Show that $2x^2 + x - 15$ can be written in the form $2(x+a)^2 + b$, where a and b are exact constants to be found. [2]

- (b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$. [2]

- (c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



- (d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions. [1]

- 3 (a) Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$

[3]

- (b) Solve the equation $\log_3 x + 3 = 10 \log_x 3$, giving your answers as powers of 3.

[4]

- 4 The polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a , b and c are integers. It is given that $p'(0) = -9$.

(a) Show that $b = -9$. [1]

It is also given that $3x + 2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c . [4]

(c) Find the quadratic $q(x)$ such that $p(x) = (3x + 2) \times q(x)$. [1]

(d) Hence find $p(x)$ as a product of linear factors with integer coefficients. [1]

- 5** A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

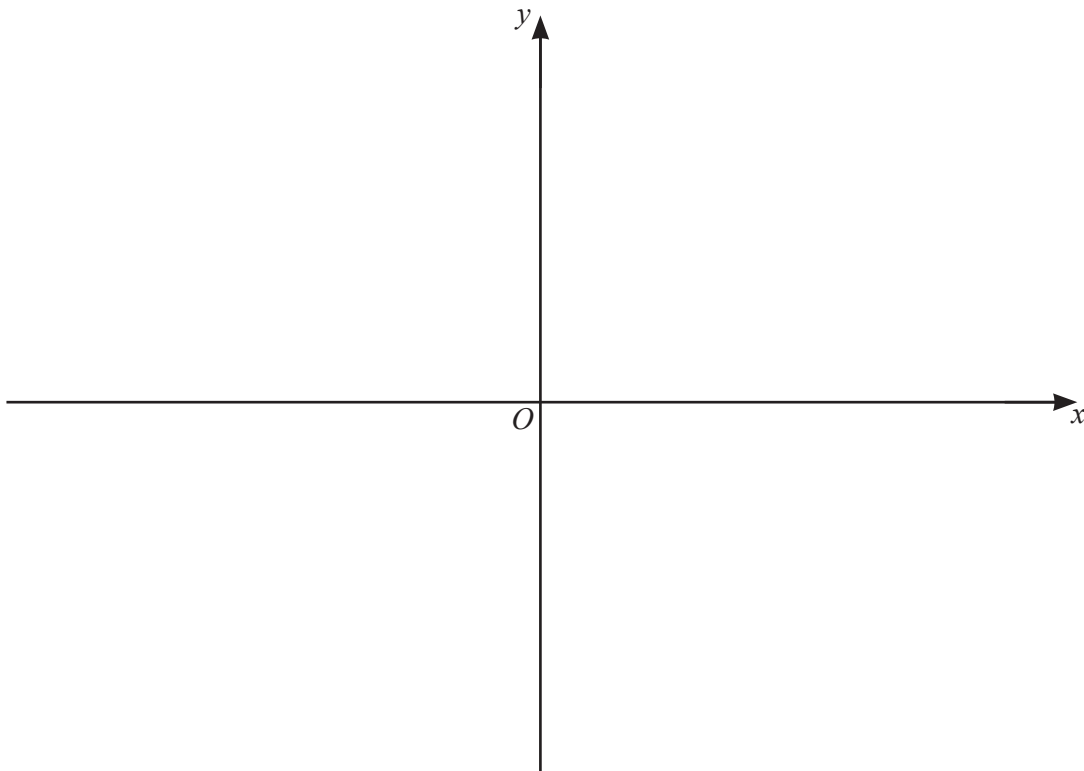
(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

6 A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f . [1]

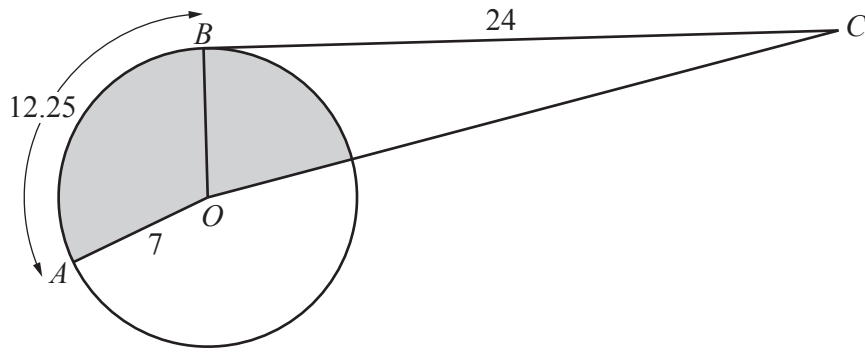
(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



- 7 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4 \sin 2x + 1) dx$. [5]

8 In this question all lengths are in metres.



The diagram shows a circle, centre O , radius 7 . The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24 . The length of the minor arc AB is 12.25 .

(a) Find the obtuse angle AOB , giving your answer in radians. [1]

(b) Find the perimeter of the shaded region. [4]

(c) Find the area of the shaded region.

[2]

9 A 6-character password is to be formed from the following characters.

Letters	A	B	C	D
Numbers	1	2	3	4
Symbols	*	#	\$	£

No character may be used more than once in any password.

(a) (i) Find the number of different 6-character passwords that can be formed.

[1]

(ii) How many of these 6-character passwords end with a symbol?

[1]

(b) Find the number of different 6-character passwords that include all the symbols, but do not start or end with a symbol.

[2]

- 10** Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where x is in radians, for $-1.5 \leq x \leq 1.5$. [5]

- 11 It is given that $\int_1^a \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x} \right) dx = \ln \frac{1}{5}$, where $a > 1$. Find the exact value of a . [6]

12 It is given that $y = \frac{(3x^2 - 2)^{\frac{2}{3}}}{x - 1}$, for $x > 1$.

(a) Write $\frac{dy}{dx}$ in the form $\frac{(3x^2 - 2)^{-\frac{1}{3}}}{(x - 1)^2}(x^2 + Ax + B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to $2 + p$, where p is small. [2]

- 13** The points P and Q have coordinates $(5, -12)$ and $(15, -6)$ respectively. The point R lies on the line l , the perpendicular bisector of the line PQ . The x -coordinate of R is 7.

(a) Find the y -coordinate of R . [4]

(b) The point S lies on l such that its distance from PQ is 3 times the distance of R from PQ . Find the coordinates of the two possible positions of S . [3]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.