



CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

0607/51

May/June 2024

1 hour 10 minutes

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

This document has **8** pages. Any blank pages are indicated.

Answer **all** the questions.

INVESTIGATION

SUMS OF NUMBERS

This investigation looks at the sum of the positive integers, $1 + 2 + 3 + \dots$.

It also looks at the sum of their cubes, $1^3 + 2^3 + 3^3 + \dots$.

1 (a) Calculate $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$.

..... [1]

(b) The numbers can also be added using this method.

Complete this method for the sum of the numbers in **part (a)**.

Step 1	Write down the first half of the numbers in a row.	1 2 3 4
Step 2	Write down the second half of the numbers underneath the first half but in reverse order.	8 7 6 5
Step 3	Add each column of two numbers to make a third row.	9 9 9 9
Step 4	Find the total of the numbers in the third row by writing the calculation as a multiplication.	$4 \times 9 = \dots\dots\dots$

[1]

2 Use the method in **Question 1(b)** to calculate the sum of the first six positive integers, $1 + 2 + 3 + 4 + 5 + 6$.

[4]

- 3 Complete the method to show that the sum of the first 30 positive integers is 465.

So $1 + 2 + 3 + \dots + 28 + 29 + 30 = 465$.

1	2	3	...	15
.....			...	16
.....			...	

..... $\times$ = 465 [3]

- 4 Find the sum of the first 160 positive integers, $1 + 2 + 3 + \dots + 160$.

..... [3]

5 Use Questions 1, 2, 3 and 4 to help you complete the table.

	Number of positive integers starting at 1	Multiplication	Sum
Question 2	4	2×5	10
	6		
Question 1	8	4×9	
Question 3	10		
	30		465
	50	25×51	1275
Question 4	128	64×129	
	160		
	204		20910

[4]

6 $1 + 2 + 3 + \cdots + n$ has n positive integers and its sum is T .

Find a formula for T in terms of n .

..... [3]

7 In the table in **Question 5** the number of integers is always even.

Does your formula in **Question 6** give the correct total when $n = 7$?

[3]

8 (a) Complete the table.

Sum of first n positive integers		Sum of first n cube numbers		
Calculation	Sum (T)		Calculation	Sum (S)
$1 + 2$	3	$1^3 + 2^3$	$1 + 8$	9
$1 + 2 + 3$		$1^3 + 2^3 + 3^3$		36
$1 + 2 + 3 + 4$			$1 + 8 + 27 + 64$	
	15	$1^3 + 2^3 + 3^3 + 4^3 + 5^3$		225

[4]

(b) (i) Write down the mathematical name for the positive integers in the Sum (S) column.

..... [1]

(ii) Write a formula for S in terms of T .

..... [1]

- (c) Use your answers to **Question 6** and **Question 8(b)** to find the sum of the cubes of the first 40 positive integers.

$$1^3 + 2^3 + 3^3 + \dots + 40^3$$

..... [4]

(d) $1^3 + 2^3 + 3^3 + \dots + n^3 = 396\,900$

Find how many cube numbers there are in this sum.

..... [4]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.