



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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0607/53

October/November 2022

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

This document has 8 pages.

Answer **all** the questions.

INVESTIGATION

TWO-STEP SEQUENCES

This investigation looks at *two-step sequences*.

These are sequences which use two steps to get from one term to the next.

The first term in every sequence is 1.

The two steps are:

- multiply by a given number
- then add a given number.

1 In this question the two steps are:

- multiply by 2
- then add 1.

$$\text{1st term} = 1$$

$$\text{2nd term} = \text{1st term} \times 2 + 1 = 1 \times 2 + 1 = 3$$

$$\text{3rd term} = \text{2nd term} \times 2 + 1 = 3 \times 2 + 1 = 7$$

$$\text{4th term} = \text{3rd term} \times 2 + 1 = 7 \times 2 + 1 = 15$$

(a) Work out the 5th term of this sequence.

..... [2]

(b) The 3rd term of this sequence is 7.
You can write 7 as $2^3 - 1$.

Complete the table.

1st term	1	$2^1 - 1$
2nd term	3	$2^2 - 1$
3rd term	7	$2^3 - 1$
4th term	15	
5th term		

[1]

- (c) Calculate the 20th term of this sequence.
Write down all the digits shown on your calculator.

..... [2]

- (d) (i) Use the last column in the table to write down an expression for the n th term of this sequence.

..... [1]

- (ii) Show that your expression gives the correct value for the 6th term of this sequence.

[2]

2 In this question the two steps are:

- multiply by 3
- then add 4.

The first term is 1.

(a) Calculate the 2nd, 3rd and 4th terms of this sequence.

1, , , [3]

(b) Complete the table.

1st term	1	$3^1 - 2$
2nd term		$3^2 - 2$
3rd term		
4th term		
5th term	241	$3^5 -$

[2]

(c) Write down an expression for the n th term of this sequence.

..... [1]

3 In this question the two steps are:

- multiply by 4
- then add 9.

The first term is 1.

Show that the expression for the n th term, $4^n - 3$, gives the correct value for the 3rd term of this sequence.

[4]

- 4 (a) Copy your results from **Question 1(d)(i)** and **Question 2(c)** into the table.

Use any patterns you notice to complete the table.

	Steps to get the next term	Expression for the n th term
Question 1(d)(i)	Multiply by 2, then add 1	
Question 2(c)	Multiply by 3, then add 4	
	Multiply by 4, then add 9	$4^n - 3$
	Multiply by, then add 16	$5^n - \dots$
	Multiply by 6, then add	$\dots - 5$
	Multiply by 7, then add 36	
	Multiply by, then add	$8^n - 7$

[4]

- (b) A sequence has the two steps that you found in the last row of the table.

Show that the expression for the n th term gives the correct value for the 3rd term of this sequence.

[3]

- (c) The n th term of a two-step sequence is $22^n - 21$.

Find the two steps.

-
- [2]

- (d) In a two-step sequence the steps are:

- multiply by 11
- then add 100.

The first term is 1.

- (i) Find the value of the term nearest to 20 000 000.
Write down all the digits shown on your calculator.

..... [2]

- (ii) Which term in the sequence is your answer to **part (i)**?

..... [1]

Question 5 is printed on the next page.

5 In this question the steps in **Question 1** are in the reverse order.

The two steps are now:

- add 1
- then multiply by 2.

- (a)** The first term is 1.
The second term is 4.

Calculate the 3rd, 4th and 5th terms.

1, 4, , , [2]

- (b)** This two-step sequence has n th term equal to $a \times 2^n - 2$.

- (i)** The first term is 1.

Use this to find the value of a .

..... [2]

- (ii)** Use **part (i)** to show that the expression for the n th term gives the correct value for the 3rd term of this sequence.

[2]

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