



CANDIDATE
NAME

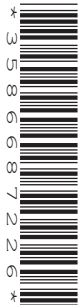
--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



0607/51

October/November 2023

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

This document has **8** pages. Any blank pages are indicated.

Answer **all** the questions.

INVESTIGATION

F-TYPE SEQUENCES

This investigation explores patterns in a special type of sequence of positive integers.

In an *F-type sequence*:

- the first two terms are any two positive integers
- after the first two terms, each term is equal to the sum of the previous two terms.

1 Here is a table of the first seven terms of an F-type sequence.

The first term F_1 is 5.

The second term F_2 is 3.

F_1	F_2	F_3	F_4	F_5	F_6	F_7
5	3	8	11	19	30	49

In the table, $F_1 + F_2 = 5 + 3 = 8 = F_3$

$F_2 + F_3 = 3 + 8 = 11 = F_4$

$F_3 + F_4 = 8 + 11 = 19 = F_5$

and so on.

(a) Calculate the 8th term (F_8) and the 9th term (F_9).

$F_8 = \dots\dots\dots$

$F_9 = \dots\dots\dots$ [3]

(b) Complete the table.

$F_1 = 5$	$F_3 - F_2 = 8 - 3 = 5$
$F_1 + F_2 = 5 + 3 = 8$	$F_4 - F_2 = \dots\dots\dots = \dots\dots\dots$
$F_1 + F_2 + F_3 = 5 + 3 + 8 = 16$	$F_5 - F_2 = \dots\dots\dots = \dots\dots\dots$
$F_1 + F_2 + F_3 + F_4 = \dots\dots\dots = \dots\dots\dots$	$\dots\dots\dots = \dots\dots\dots = 27$

[5]

- (c) Use what you notice in the table in **part (b)** to complete this statement.

$$F_1 + F_2 + F_3 + F_4 + F_5 = F_{\dots\dots\dots} - F_{\dots\dots\dots} \quad [1]$$

- (d) Use the statement in **part (c)** to complete this general statement.

$$F_1 + F_2 + F_3 + \dots + F_n = F_{n+\dots\dots\dots} - F_{\dots\dots\dots} \quad [1]$$

- (e) Show that your statement in **part (d)** is correct when $n = 7$.

[3]

2 In another F-type sequence the first term is 3 and the second term is 1.

(a) Complete the first five terms.

3, 1,,, [2]

(b) Is your statement in **Question 1(c)** correct for the sum of the first five terms in this sequence?

..... [3]

3 In another F-type sequence the 2nd term is 3 and the 12th term is 652.

Use your answer to **Question 1(d)** to find the sum of the first 10 terms.

..... [2]

4 In another F-type sequence the first term is 6 and the second term is k .

- (a) Find an expression, in terms of k , for the 4th term.
Write your answer in its simplest form.

..... [2]

- (b) The 4th term of this sequence is 14.

Find k and complete the first four terms.

6,,, 14 [3]

5 An F-type sequence has five terms.

(a) There is a relationship between the middle term and the sum of the 1st term and the 5th term.

Investigate this relationship by making up at least three numerical examples of F-type sequences.
Write down this relationship.

..... [6]

(b) In another F-type sequence the first term is x and the second term is y .

(i) Find, in its simplest form, an expression in terms of x and y for the 5th term.

..... [3]

(ii) Use algebra to show that the relationship in **part (a)** is correct.

[2]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.