

CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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4037/12

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

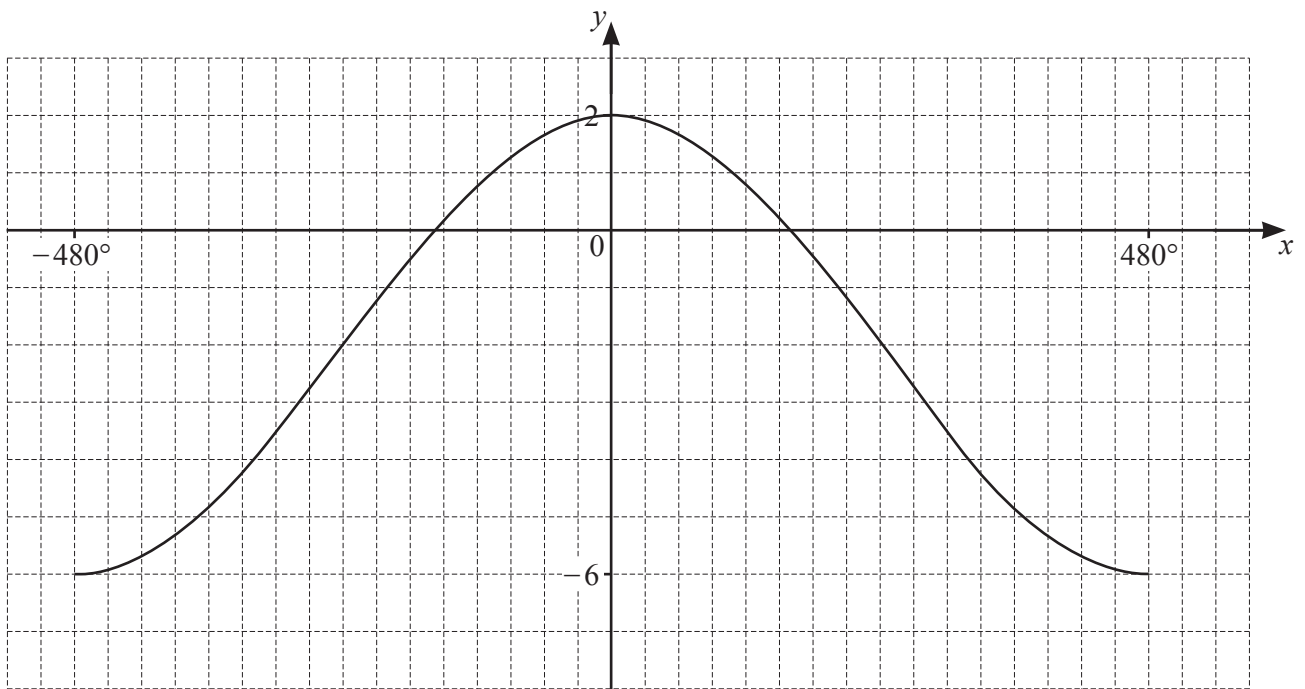
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

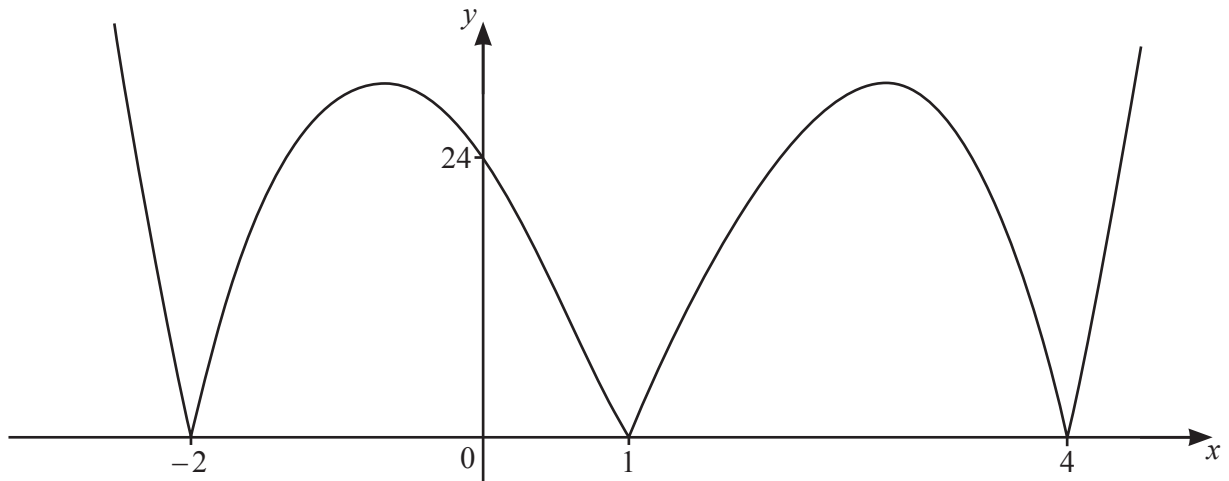
- 1 The diagram shows the graph of $y = a \cos bx + c$. Find the values of the constants a , b and c . [3]



2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(2 + \sqrt{5})x^2 = 4x + 3(2 - \sqrt{5})$, giving your answers in the form $a + b\sqrt{5}$ where a and b are integers. [5]

3 (a)

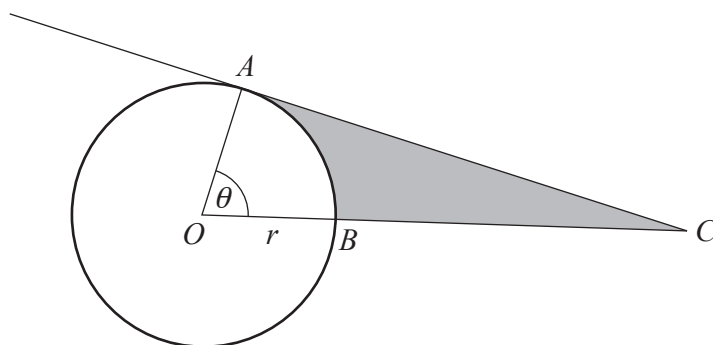


The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic polynomial. Find, in factorised form, the possible expressions for $f(x)$. [3]

(b) Solve the inequality $|5x - 2| \leq |4x + 1|$.

[4]

- 4 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius r . The points A and B lie on the circumference of the circle such that the angle AOB is θ and the length of the minor arc AB is 12. The area of the minor sector AOB is 57.6 cm^2 . The point C lies on the tangent to the circle at A such that OBC is a straight line.

- (a) Find the values of r and θ .

[4]

- (b) Find the area of the shaded region. Give your answer correct to 1 decimal place.

[3]

- 5 (a) Find the exact solutions of the equation $6p^{\frac{1}{3}} - 5p^{-\frac{1}{3}} - 13 = 0$. [4]

- (b) Solve the equation $2\lg(2x+5) - \lg(x+2) = 1$, giving your answers in exact form. [6]

- 6 (a) Given that $\cot^2 \theta = \frac{1}{y+2}$ and $\sec \theta = x-4$, find y in terms of x . [2]

- (b) Solve the equation $\sqrt{3} \operatorname{cosec}\left(2\phi + \frac{3\pi}{4}\right) = 2$, for $-\pi < \phi < \pi$, giving your answers in terms of π . [5]

- 7 (a) Find the number of ways in which 14 people can be put into 4 groups containing 2, 3, 4 and 5 people. [3]

- (b) 6-digit numbers are to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit may be used only once in any 6-digit number. A 6-digit number must not start with 0. Find how many 6-digit numbers can be formed if

(i) there are no further restrictions [1]

(ii) the 6-digit number is divisible by 10 [1]

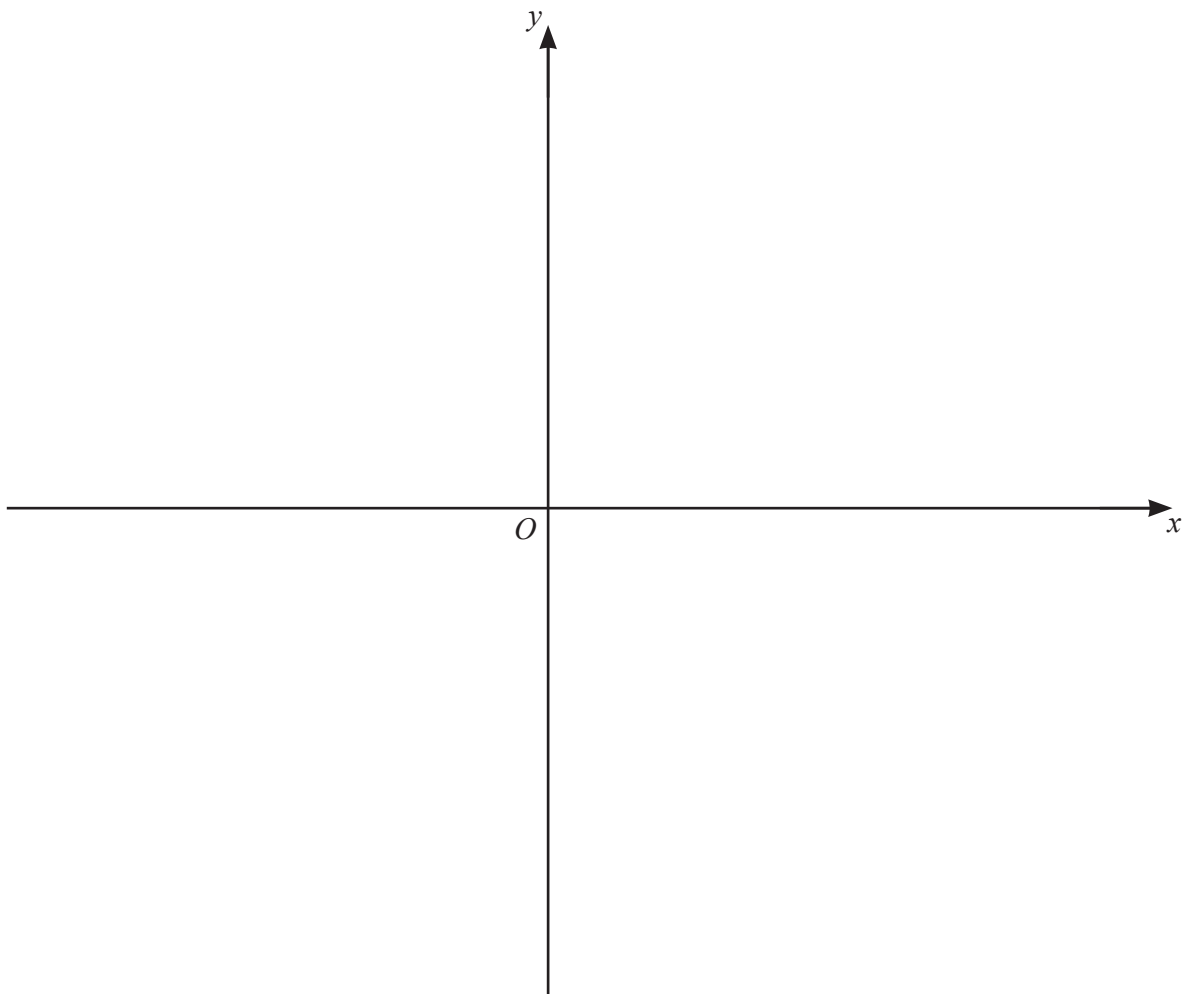
(iii) the 6-digit number is greater than 500 000 and even. [3]

8 It is given that $f(x) = 2 \ln(3x - 4)$ for $x > a$.

(a) Write down the least possible value of a . [1]

(b) Write down the range of f . [1]

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]

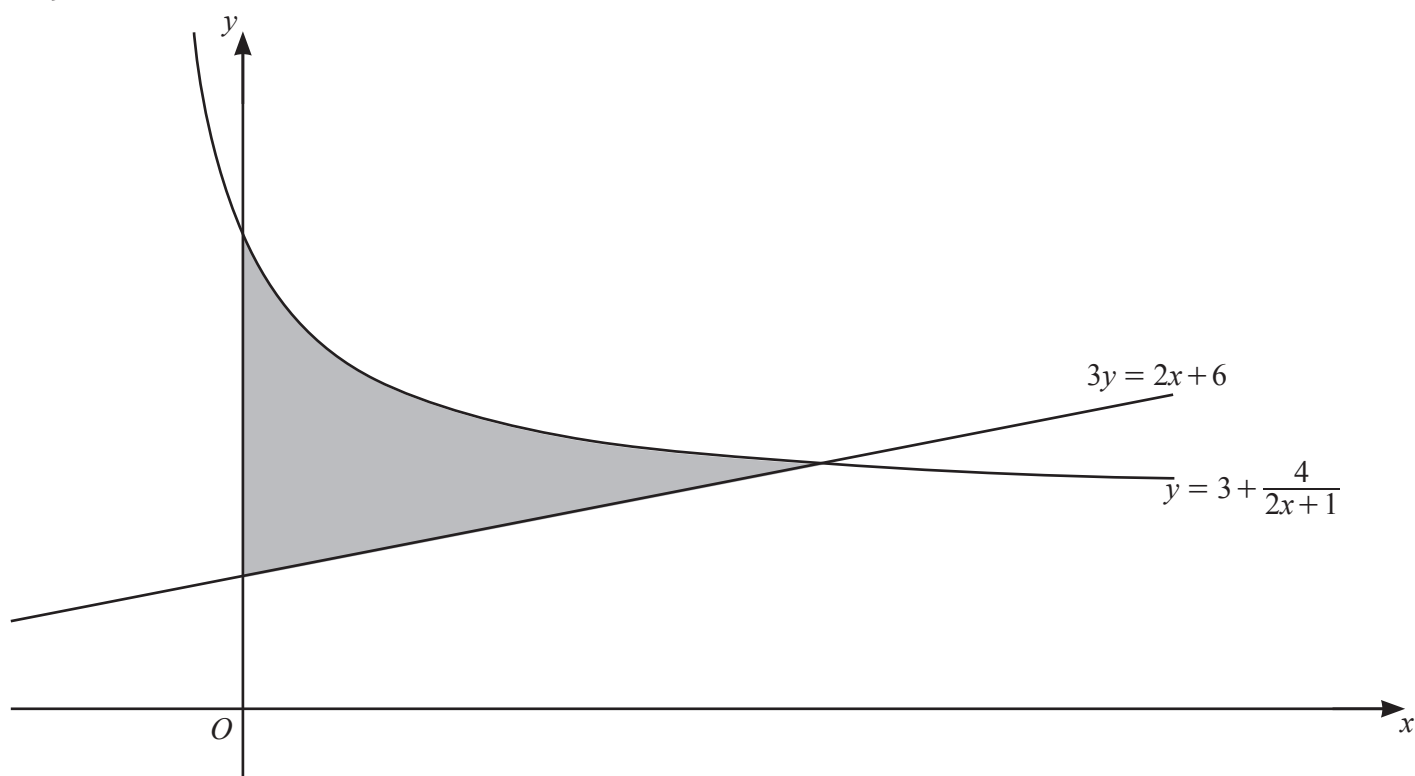


It is given that $g(x) = 2x - 3$ for $x \geq 3$.

(d) (i) Find an expression for $g(g(x))$. [1]

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

9



The diagram shows part of the curve $y = 3 + \frac{4}{2x+1}$ and the straight line $3y = 2x + 6$. Find the area of the shaded region, giving your answer in exact form. [10]

Continuation of working space for Question 9.

10 (a) The first three terms of an arithmetic progression are $(2x + 1)$, $4(2x + 1)$ and $7(2x + 1)$, where $x \neq -\frac{1}{2}$.

(i) Show that the sum to n terms can be written in the form $\frac{n}{2}(2x + 1)(An + B)$, where A and B are integers to be found. [2]

(ii) Given that the sum to n terms is $(54n + 37)(2x + 1)$, find the value of n . [2]

(iii) Given also that the sum to n terms in **part (ii)** is equal to 1017.5, find the value of x . [2]

- (b) The first three terms of a geometric progression are $(2y+1)$, $3(2y+1)^2$ and $9(2y+1)^3$, where $y \neq -\frac{1}{2}$.

Given that the n th term of the progression is equal to 4 times the $(n+2)$ th term, find the possible values of y , giving your answers as fractions. [4]

- (c) The first three terms of a different geometric progression are $\sin \theta$, $2 \sin^3 \theta$ and $4 \sin^5 \theta$, for $0 < \theta < \frac{\pi}{2}$. Find the values of θ for which the progression has a sum to infinity. [3]

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