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4037/22

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Solve the inequality $3x^2 - 12x + 16 > 3x + 4$. [3]

- (b) (i) Write $3x^2 - 12x + 16$ in the form $a(x + b)^2 + c$ where a , b and c are integers. [3]

- (ii) Hence, write down the equation of the tangent to the curve $y = 3x^2 - 12x + 16$ at the minimum point of the curve. [1]

2 A curve has equation $y = 32x^2 + \frac{1}{8x^2}$ where $x \neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

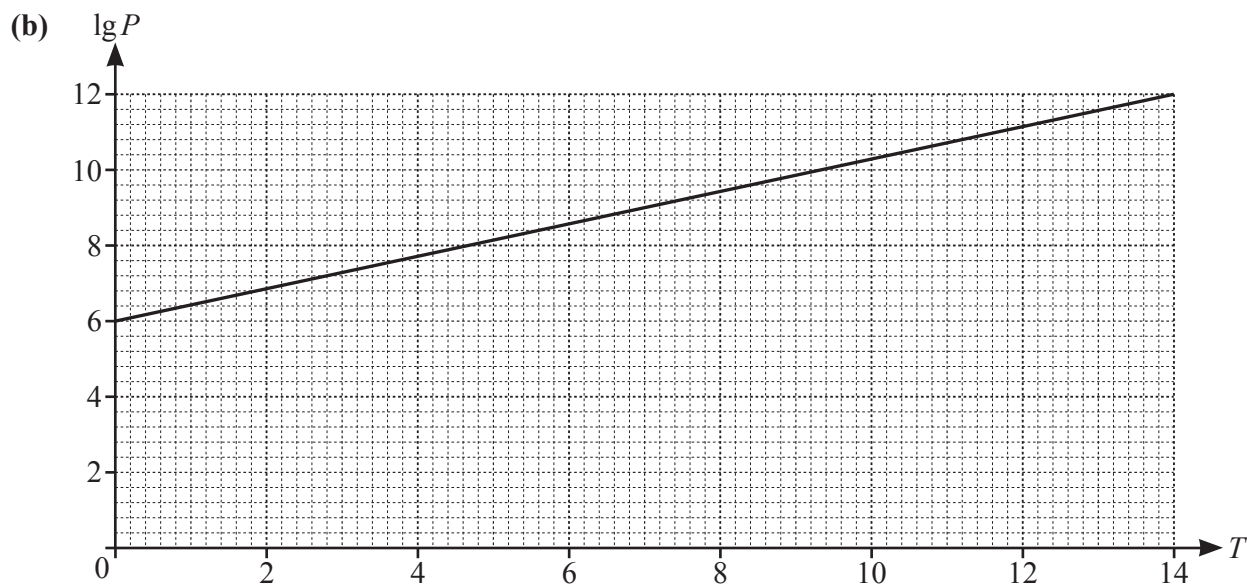
(a) Show that $x + 3$ is a factor of $-12 + 23x + 3x^2 - 2x^3$. [1]

(b) The curve $y = -5 + 33x + 3x^2 - 2x^3$ and the line $y = 10x + 7$ intersect at three points, A , B and C . These points are such that the x -coordinate of A has the least value and the x -coordinate of C has the greatest value. Show that B is the mid-point of AC . [7]

- 4 Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leq x \leq \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

- 5 Variables P and T are known to be connected by the relationship $P = Ab^T$, where A and b are constants. Values of P are found for certain values of time, T .

(a) Show that a graph of $\lg P$ against T will be a straight line. [2]



The diagram shows the graph of $\lg P$ against T . The graph passes through $(0, 6)$ and $(14, 12)$. Find the values of A and b . [4]

- (c) Using the graph or otherwise, find the length of time for which P is between 100 million and 1000 million. [3]

- 6 (a) (i) Find the first three terms in the expansion of $\left(1 + \frac{x}{7}\right)^5$, in ascending powers of x . Simplify the coefficient of each term. [2]

- (ii) The expansion of $7(1+x)^n\left(1 + \frac{x}{7}\right)^5$, where n is a positive integer, is written in ascending powers of x . The first two terms in the expansion are $7 + 89x$. Find the value of n . [2]

- (b) In the expansion of $(k - 2x)^8$, where k is a constant, the coefficient of x^4 divided by the coefficient of x^2 is $\frac{5}{8}$. The coefficient of x is positive. Form an equation and hence find the value of k . [5]

7 (a) $f(x) = \sqrt{3 + (4x - 2)^5}$ where $x > 1$.

Find an expression for $f'(x)$, giving your answer as a simplified algebraic fraction. [3]

(b) Variables x and y are related by the equation $y = \frac{5x}{3x+2}$. Using differentiation, find the approximate change in x when y increases from 10 by the small amount 0.01. [4]

(c) (i) Differentiate $y = x^3 \ln x$ with respect to x . [2]

(ii) Hence find $\int \left(\frac{x^2}{6} (2 + 3 \ln x) \right) dx$. [3]

- 8 A curve has equation $y = \cos \frac{x}{4}$ where x is in radians. The normal to the curve at the point where $x = \frac{4\pi}{3}$ cuts the x -axis at the point P . Find the exact coordinates of P . [7]

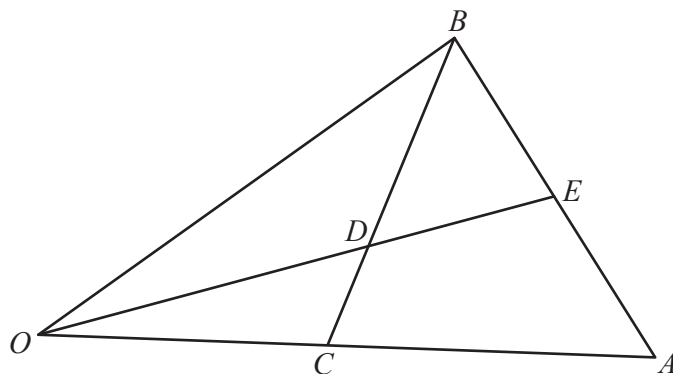
- 9 A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v \text{ ms}^{-1}$, is given by

$$v = e^{\frac{t}{4}} \quad \text{for } 0 \leq t \leq 4,$$

$$v = \frac{16e}{t^2} \quad \text{for } 4 \leq t \leq k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k .
[6]

10



The diagram shows a triangle OAB . The point C is the mid-point of OA . The point D lies on CB such that $CD : DB = 2 : 3$.

$$\overrightarrow{OC} = \mathbf{c} \quad \overrightarrow{CB} = \mathbf{b}$$

The point E lies on AB such that $\overrightarrow{OE} = \lambda \overrightarrow{OD}$ and $\overrightarrow{AE} = \mu \overrightarrow{AB}$ where λ and μ are scalars. Find two expressions for $\overrightarrow{OE}$, each in terms of $\mathbf{b}$, $\mathbf{c}$ and a scalar, and hence find $AE : EB$.

[8]

Continuation of working space for Question 10.

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