



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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4037/21

May/June 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

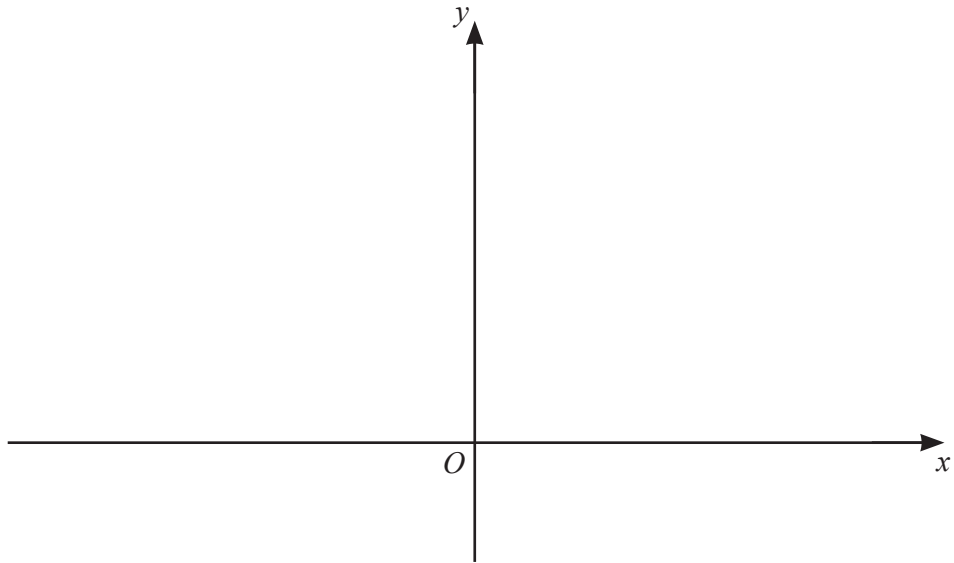
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graph of $y = |4x - 6|$, showing the points where the graph meets the axes. [2]



- (b) Solve the equation $|4x - 6| = |2x|$. [3]

2 (a) Write $3 + 4x - 2x^2$ in the form $a + b(x + c)^2$, where a , b and c are integers. [3]

(b) Hence write down the range of the function $f(x) = 3 + 4x - 2x^2$, where $x \in \mathbb{R}$. [1]

3 Use algebra to show that the equation $5x(x - 3) = 5x - 26$ has no real solutions. [3]

4 DO NOT USE A CALCULATOR IN THIS QUESTION.

- (a) Find the exact distance between the two points where the curve $9(x-1)^2 + 4(y-3)^2 = 36$ cuts the y -axis. [4]
- (b) Find the coordinates of the points where the curve with equation $2x^2 + 83xy = x^3y - 20x$ intersects the curve with equation $y = \frac{1}{x}$. Give each of your answers in the form $a + b\sqrt{c}$, where a and b are rational and c is the smallest integer possible. [6]

5 There are 3 women, 2 men and 4 children in a choir.

(a) The choir stands in a single straight line.

(i) Find the number of possible arrangements if the first person and last person are both women. [2]

(ii) Find the number of possible arrangements if all the children stand next to each other. [2]

(b) Four of the choir are selected to sing in a group.

(i) Find the number of different selections if no man is chosen. [2]

(ii) Find the number of different selections if at least 2 women are chosen. [2]

- 6 Variables x and y are such that $y = \cos x \sin^2 x$. Use differentiation to find the approximate change in y as x increases from 3 to $3 + h$, where h is small. [5]

- 7 It is given that $y = mx^2 + \frac{x}{2} + n$, where m and n are non-zero constants. It is also given that $3\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^2 - y$ for all values of x . Find the values of m and n . [4]

- 8 (a) In an arithmetic progression, the sum of the first 30 terms is -1065 .
The sum of the **next** 20 terms is -2210 .
Find the first term and the common difference.

[5]

- (b) A geometric progression is such that the first term is 4 and the sum of the first three terms is 7. Find the two possible values of the common ratio and find the sum to infinity for the convergent progression. [5]

- 9 The functions f and g are defined by

$$f(x) = \frac{3x^2}{4x-1} \quad \text{for } x < 0$$
$$g(x) = \frac{1}{x^2} \quad \text{for } x < 0.$$

- (a) Explain why the function fg does **not** exist. [1]

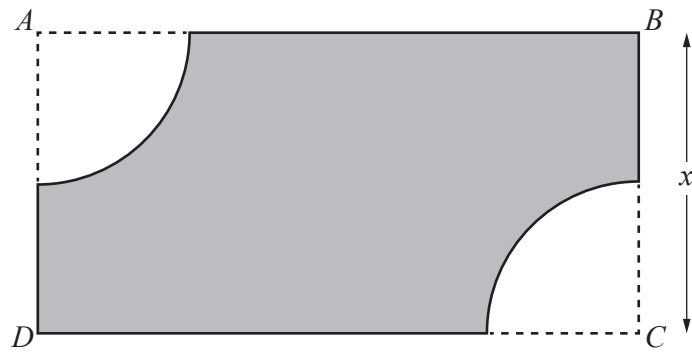
- (b) Given that the function gf does exist, find and simplify an expression for $gf(x)$. [2]

- (c) Show that $f^{-1}(x)$ can be written as $\frac{px - \sqrt{x(qx+r)}}{3}$ where p , q and r are integers. [4]

10 (a) Show that $(\tan x + \sec x)^2$ can be written as $\frac{1 + \sin x}{1 - \sin x}$. [4]

(b) Hence solve the equation $(\tan 3\theta + \sec 3\theta)^2 = 6$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

11 In this question all lengths are in centimetres.



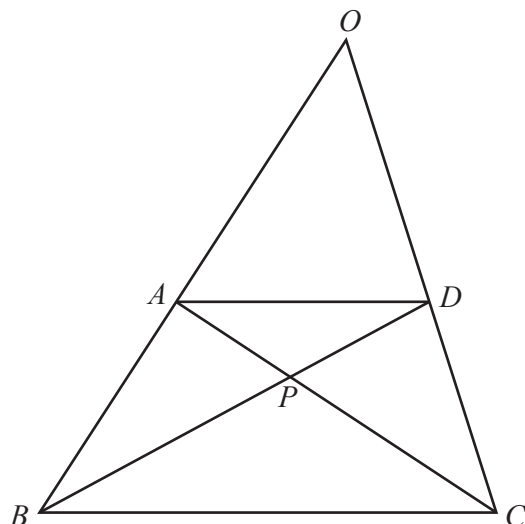
The diagram shows a rectangle $ABCD$ with $BC = x$.
The area of the rectangle is 400 cm^2 .

Two identical quarter-circles of radius $\frac{x}{2}$, with centres A and C , are removed from the rectangle to make the shaded shape.

Given that x can vary, find the value of x that gives the minimum value of the perimeter of the shaded shape and hence find this minimum value. [7]

Continuation of working space for Question 11.

12



The diagram shows a triangle OBC .

$OA : OB = 4 : 7$ and $OD : OC = 4 : 7$.

$$\overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

The point P is the point of intersection of AC and BD such that $\overrightarrow{AP} = \lambda \overrightarrow{AC}$ and $\overrightarrow{BP} = \mu \overrightarrow{BD}$ where λ and μ are scalars.

- (a) Find two expressions for $\overrightarrow{OP}$, each in terms of $\mathbf{b}$, $\mathbf{c}$ and a scalar, and hence show that P divides both AC and DB in the ratio $4 : 7$. [7]

- (b) The point Q is such that $\overrightarrow{OQ} = \frac{2}{7}\mathbf{b} + \frac{2}{7}\mathbf{c}$.

Use a vector method to show that O , Q and P are collinear. Justify your answer.

[2]

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