

Cambridge O Level

ADDITIONAL MATHEMATICS**4037/22**

Paper 2

May/June 2026**MARK SCHEME**Maximum Mark: 80

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **33** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Accuracy mark awarded two
	Accuracy mark awarded three
	Independent mark awarded zero
	Independent mark awarded one
	Independent mark awarded two
	Independent mark awarded three
	Benefit of the doubt
	Communication mark

Annotation	Meaning
	Incorrect
FT	Follow through
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
M3	Method mark awarded three
MR	Misread
O	Omission
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
Pre	Premature rounding/approximation
SC	Special case
SEEN	Indicates that work/page has been seen
TE	Transcription error
	Correct

Annotation	Meaning
	Correct answer from incorrect working

PUBLISHED**MARK SCHEME NOTES**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

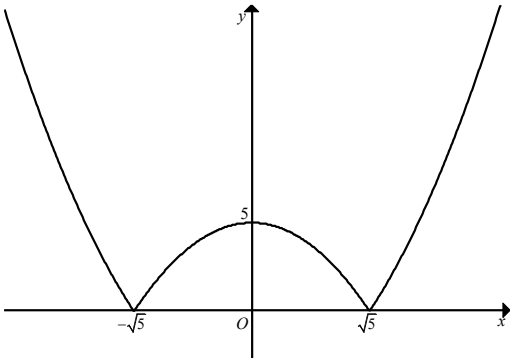
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks	Guidance
1(a)	$[y =] (x - 3)^2 + 1$	M1		• Condone $(x - 3)^2 - 3^2 + 10$ followed by the correct answer for 2 marks • Allow corrected ‘slips’ when completing the square if they show an attempt at their method
	(3, 1)	A1	If 0 scored, SC1 for (3, 1) from a completed square form with a minor slip e.g. $(x - 3) + 1$ or $(x - 3)^2 + 1 = 0$	• Must be from completing the square • Condone omission of brackets • Accept $x = 3, y = 1$ • Accept use of $y = 0$ in the method for completing the square for 2 marks if they find $(x - 3)^2 + 1$ and state it without ‘=0’ otherwise SC1 only.
1(b)	$[\text{gradient } AC =] \frac{10 - \text{their } 1}{0 - \text{their } 3} \text{ oe or } -3$	M1	FT <i>their</i> (3, 1) however obtained	• If FT evidence of a gradient calculation must be seen e.g. $\frac{a}{b}$ where $a = 10 - \text{their } 1$ $b = [0] - \text{their } 3$ or $a = \text{their } 1 - 10$ $b = \text{their } 3[-0]$ • Accept e.g. $1 = 3m + 10$ • M0 if they then go on to find the perpendicular gradient
	$y - 7 = -3(x - 6) \text{ oe, isw}$	A1	FT <i>their</i> gradient AC	• Any correct form e.g. • $y = 25 - 3x$ or $\frac{y - 7}{x - 6} = -3$ • Must be a complete equation e.g. • M1 A0 for $y = -3x + c$ and $c = 25$ only

Question	Answer	Marks	Partial Marks	Guidance
2	$(2k + 3)^2 - 4(k - 2)(3)$ [*0 where * is = or any inequality sign] oe	M1		Allow e.g. use of $(2k - 3)^2 - 4(k - 2)(3)$ to be marked as a miscopy.
	$4k^2 + 33$ [*0]	A1		Accept e.g. $k^2 = \frac{-33}{4}$
	Correct argument e.g. $4k^2 + 33 = 0$ is not possible oe or $4k^2 + 33 = 0$ so k cannot be real oe	A1		- Must not be contradicted - Must see or clearly imply ‘= 0’ - The ‘= 0’ may be implied in the final argument even if they have used e.g. > 0 in the earlier steps - No FT on an incorrect expression in k.
3(a)	$\left[\frac{dy}{dx} = \right] \frac{1}{2}(x+1)^{-\frac{1}{2}} - 5\sin(5x+3)$ oe, isw	3	M1 for $\frac{1}{2}(x+1)^{-\frac{1}{2}}$ M2 for $-5\sin(5x+3)$ or M1 for $k\sin(5x+3)$ where k is negative or 5 or $-5\sin 5x+3$ or SC1 for $-5\sin(5x+3)+c$ maximum of 2 marks if not in a single expression	- For 3 marks the derivatives must be combined in a single expression; for partial marks the derivatives may be separate or could be one of two terms in a single expression $\frac{1}{2}(x+1)^{-\frac{1}{2}} - 5\sin(5x+3) + c$ earns M1 SC1 as the inclusion of $+c$ destroys the M2 - In $k\sin(5x+3)$, k can be any negative value including -1 - Allow maximum of 2 marks for correct derivatives embedded in e.g. product rule, provided it is clear.

Question	Answer	Marks	Partial Marks	Guidance
3(b)	$3 \tan \frac{x}{3} + c$ isw	3	M1 for $k \tan \frac{x}{3}$ A1 for $3 \tan \frac{x}{3}$ or $\frac{1}{1} \tan \frac{x}{3} + c$	

Question	Answer	Marks	Partial Marks	Guidance
4(a)	Correct smooth curve with intercepts indicated 	B3	B2 for correct smooth curve with - Two cusps on the x-axis roughly equidistant from O; - local maximum on the y-axis; - middle section with correct curvature - correct curvature at outer branches or B1 for correct smooth curve with three of these four properties B1 for intercepts 5, $-\sqrt{5}$ and $\sqrt{5}$ or appropriately indicated; must have attempted correct smooth curve maximum of 2 marks if not fully correct	For shape B2: - Allow imperfect scale and symmetry - Ignore middle section below axis if also correct above Outer branches may be somewhat straight BUT must be an attempt at a smooth curve - No shape marks if any <u>clearly</u> ruled sections At each end: condone a little flicking but must not approach a turning point For intercepts B1: - If coordinates and intercepts stated and they disagree, mark the better; - If coordinates only, must be correct - Accept decimals to 3 or more sf e.g. 2.24 or 2.236[06...] - Coordinates of intercepts may be stated at the side or below the graph, but they must clearly be the intercepts and not just coordinates in a table of points to plot, for example - The shape mark(s) may not have been earned, but must be a recognisable attempt at the modulus of a quadratic

Question	Answer	Marks	Partial Marks	Guidance
4(b)	EITHER Solution 1 Interprets the modulus and uses at least one value of x to form a pair of linear equations e.g. using 4 and -4 $2.5a + b = 4$ and $-1.5a + b = -4$ or $2.5a + b = 4$ and $2.5a + b = -4$ or $-1.5a + b = 4$ and $2.5a + b = -4$ or $-1.5a + b = 4$ and $-1.5a + b = -4$ oe	(B1)	OR uses at least one value of x to form a quadratic equation e.g. $(2.5a + b)^2 = 16$ or $(-1.5a + b)^2 = 16$ or $(2.5a + b)^2 = (-1.5a + b)^2$	- May be in a list of 4 equations - May be e.g. $2.5a + b = 4$ and $1.5a - b = 4$ $2.5a + b = \pm(-1.5a + b)$
	Uses two valid equations to correctly eliminate one unknown at least once e.g. $2.5a + 1.5a - 4 = 4$ or $4a = 8$ or $4a = -8$ or $(2.5(-2b) + b)^2 = 16$	M1		Valid equations e.g. $2.5a + b = 4$ and $-1.5a + b = -4$ oe or $2.5a + b = -4$ and $-1.5a + b = 4$ oe or $(2.5a + b)^2 = 16$ and $(-1.5a + b)^2 = 16$ oe
	$a = 2, b = -1$ or $a = -2, b = 1$ soi or $a = \pm 2$ or $b = \pm 1$	A1		- Accept a correct expression: $2x - 1$ or $-2x + 1$ - Accept embedded e.g. $f(x) = 2x - 1$
	$2x - 1$ and $-2x + 1$ and no others	A1)		- Condone $a = 2, b = -1$ and $a = -2, b = 1$ - Correct expressions without working or without much working imply 4 marks

Question	Answer	Marks	Partial Marks	Guidance
4(b)	OR Solution 2 Vertex of $y = f(x) $: (0.5, 0)	(B1)	OR statement of a relevant pair of points e.g. (2.5, 4), (−1.5, −4) or (2.5, −4), (−1.5, 4)	Points may be embedded in e.g. equations
	Finds gradient of line segment joining (0.5, 0) and (2.5, 4) as 2 or (0.5, 0) and (−1.5, 4) as −2	M1	OR finds gradient of line segment joining (2.5, 4), (−1.5, −4) as 2 or (2.5, −4), (−1.5, 4) as −2	Does not have to be identified as a
	$a = 2, b = -1$ or $a = -2, b = 1$ so or $a = \pm 2$ or $b = \pm 1$	A1		- Accept a correct expression: $2x - 1$ or $-2x + 1$ - Accept embedded e.g. $f(x) = 2x - 1$
	$2x - 1$ and $-2x + 1$ and no others	A1)		- Condone $a = 2, b = -1$ and $a = -2, b = 1$ - Correct expressions without working or without much working imply 4 marks

Question	Answer	Marks	Partial Marks	Guidance
5(a)	68 880	3	M2 for $41 \times 8 \times 7 \times 6 \times 5 \times 1$ oe or a correct sum or difference of calculations or values e.g.	M1 for 53760 could be also earned using All arrangements ending 2, 4, 6, 8 – those starting with 0 and ending 2, 4, 6, 8 $9 \times 8 \times 7 \times 6 \times 5 \times 4 - 1 \times 8 \times 7 \times 6 \times 5 \times 4$ or ${}^9P_5 \times 4 - {}^8P_4 \times 4$ or $60480 - 6720$ M1 for the 26880 in ${}^{10}P_6 - 26880$
			OR ends 2, 4, 6 or 8 + ends 0: $8 \times 8 \times 7 \times 6 \times 5 \times 4 + 9 \times 8 \times 7 \times 6 \times 5 \times 1$ oe	M2 for e.g. $53760 + 15120$ or $8 \times {}^8P_4 \times 4 + 9 \times {}^8P_4$ or $8 \times {}^8C_4 \times 24 \times 4 + 9 \times {}^8C_4 \times 24$
			OR starts 2, 4, 6 or 8 + starts 1, 3, 5, 7 or 9: $4 \times 8 \times 7 \times 6 \times 5 \times 4 + 5 \times 8 \times 7 \times 6 \times 5 \times 5$ oe	M2 for e.g. $26880 + 42000$ or $4 \times {}^8P_4 \times 4 + 5 \times {}^8P_4 \times 5$ or $4 \times {}^8C_4 \times 24 \times 4 + 5 \times {}^8C_4 \times 24 \times 5$
			OR all arrangements – starts 0 – odd: $10 \times 9 \times 8 \times 7 \times 6 \times 5 - 1 \times 9 \times 8 \times 7 \times 6 \times 5 - 8 \times 8 \times 7 \times 6 \times 5 \times 5$	M2 for e.g. $151200 - 15120 - 67200$ or ${}^{10}P_6 - {}^9P_5 - 8 \times {}^8P_4 \times 5$ or $136080 - 67200$ or $84000 - 15120$
			OR all even arrangements – even that start 0 $9 \times 8 \times 7 \times 6 \times 5 \times 5 - 1 \times 8 \times 7 \times 6 \times 5 \times 4$	M2 for e.g. $75600 - 6720$ or $9 \times {}^8P_4 \times 5 - 1 \times {}^8P_4 \times 4$
			or M1 for any one correct calculation or evaluation of e.g.	For M1 allow embedded as part of a sum or difference but not a product
			ends 2, 4, 6 or 8: $8 \times 8 \times 7 \times 6 \times 5 \times 4$ oe	e.g. 53760 or $8 \times {}^8P_4 \times 4$ or $8 \times {}^8C_4 \times 24 \times 4$
			or starts or ends 0: $9 \times 8 \times 7 \times 6 \times 5 \times 1$ oe	e.g. 15120 or $9 \times {}^8P_4$ or $9 \times {}^8C_4 \times 24$ or 9P_5

Question	Answer	Marks	Partial Marks	Guidance
5(a)			or starts 2, 4, 6 or 8: $4 \times 8 \times 7 \times 6 \times 5 \times 4$ oe	e.g. 26880 or $4 \times {}^8P_4 \times 4$ or $4 \times {}^8C_4 \times 24 \times 4$
			or starts 1, 3, 5, 7 or 9: $5 \times 8 \times 7 \times 6 \times 5 \times 5$ oe	e.g. 42000 or $5 \times {}^8P_4 \times 5$ or $5 \times {}^8C_4 \times 24 \times 5$
			or all arrangements – odd: $10 \times 9 \times 8 \times 7 \times 6 \times 5 - 8 \times 8 \times 7 \times 6 \times 5 \times 5$ oe	e.g. 84000 or $151200 - 67200$ or ${}^{10}P_6 - 8 \times {}^8P_4 \times 5$
			or all arrangements–starts 0: $10 \times 9 \times 8 \times 7 \times 6 \times 5 - 1 \times 9 \times 8 \times 7 \times 6 \times 5$ oe	e.g. 136080
			or all even arrangements $9 \times 8 \times 7 \times 6 \times 5 \times 5$	e.g. 75600 or ${}^9P_5 \times 5$ or $9 \times {}^8P_4 \times 5$
			or even that start 0 $1 \times 8 \times 7 \times 6 \times 5 \times 4$	e.g. 6720 or $1 \times {}^8P_4 \times 4$
5(b)	6468	3	M2 for ${}^9C_4 \times {}^7C_6 + {}^9C_5 \times {}^7C_5 + {}^9C_6 \times {}^7C_4$ oe OR ${}^{16}C_{10} - \left({}^9C_3 \times {}^7C_7 + {}^9C_7 \times {}^7C_3 + {}^9C_8 \times {}^7C_2 + {}^9C_9 \times {}^7C_1 \right)$ or M1 for any two correct products or any two of the three values in the sum or all three products or all three values listed but not summed OR for ${}^{16}C_{10}$ minus the sum of at least two correct products or two correct values out of three	$(126 \times 7) + (126 \times 21) + (84 \times 35)$ or 882 + 2646 + 2940 $8008 - (84 + 36 \times 35 + 9 \times 21 + 7)$ or 8008 – (84 + 1260 + 189 + 7) or 8008 – 1540

Question	Answer	Marks	Partial Marks	Guidance
6	$a^4 + 4a^3x + 6a^2x^2$ soi and $1 + 5ax + 10a^2x^2$ soi	2	B1 for each expression	Implied by correct equations or a full set of correctly
	$a = \pm 2$	2	M1 for $a^4 = 16$ soi	- Must be free of x for obtained correct values or sight of $16 + 32x + 24x^2$ and $1 + 10x + 40x^2$ or $a = 2$, $b = 192$ and $c = 984$ - Terms may be listed rather than summed - Ignore extra terms - Accept e.g. ${}^4C_0 \times a^4 \times x^0 + {}^4C_1 \times a^3 \times x + {}^4C_2 \times a^2 \times x^2$ and ${}^5C_0 [\times 1] \times (ax)^0 + {}^5C_1 [\times 1] \times ax + {}^5C_2 [\times 1] \times a^2x^2$ M1 - Any binomial coefficients must be evaluated for the M1
	$[a = 2] \quad b = 192$ $[a = -2] \quad b = -192$	2	M1 FT for $b = 5a^5 + 4a^3$ soi; FT <i>their</i> integer value of a	- M1 implied by e.g. 192 or -192 or $160 + 32$ or $-160 - 32$ - Must be free of x for M1 - Any binomial coefficients must be evaluated for the M1 - If FT substitution of <i>their</i> integer value of a into $b = 5a^5 + 4a^3$ must be explicitly seen e.g. $a = 4$ and $b = 5 \times 4^5 + 4 \times 4^3$

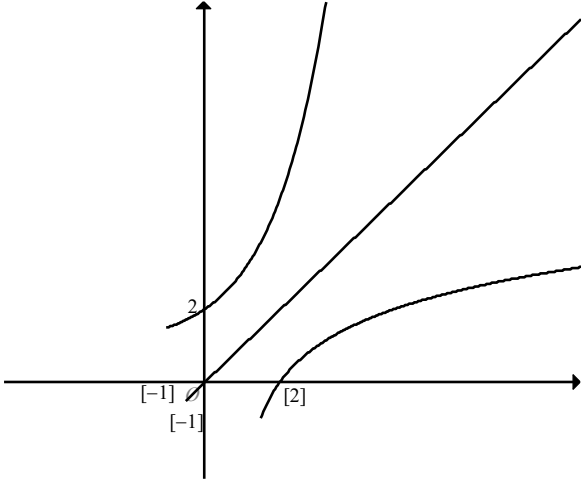
Question	Answer	Marks	Partial Marks	Guidance
6	$c = 984$	2	M1 FT for $c = 10a^6 + 20a^4 + 6a^2$ soi; FT <i>their</i> integer value of a	- May have come from use of e.g. $a = 2$ only (→maximum of 6 marks if $a = 2$ only) - Implied by e.g. $24 + 640 + 320$ - Must be free of x for M1 - Any binomial coefficients must be evaluated for the M1 - If FT substitution of <i>their</i> integer value of a into $c = 10a^6 + 20a^4 + 6a^2$ must be explicitly seen e.g. $a = 4$ and $c = 10 \times 4^6 + 20 \times 4^4 + 6 \times 4^2$
7			All through question 7, allow the use of correctly written column vectors but not e.g. $\begin{pmatrix} a \\ b \end{pmatrix}$	
7(a)	$18\mathbf{i} + 21\mathbf{j}$ oe, isw	1		Allow unsimplified e.g. $2\mathbf{i} + 5\mathbf{j} + 2(8\mathbf{i} + 8\mathbf{j})$ isw
7(b)	$2\mathbf{i} + 5\mathbf{j} + t(8\mathbf{i} + 8\mathbf{j})$ oe, isw	1		Accept e.g. $\begin{pmatrix} 2+8t \\ 5+8t \end{pmatrix}$

Question	Answer	Marks	Partial Marks	Guidance
7(c)	$20\mathbf{i} + 8\mathbf{j} + 2\left(-7.5\mathbf{i} + \left(\frac{5\sqrt{3}}{2}\right)\mathbf{j}\right)$ oe, isw or $5\mathbf{i} + (8 + 5\sqrt{3})\mathbf{j}$	B3	B2 for the two components of the velocity vector $x = -7.5$ and $y = \frac{5\sqrt{3}}{2}$ or 4.33[01...] oe, soi or B1 for each correct component If 0 scored, SC1 for reversed components $y = -7.5$ and $x = \frac{5\sqrt{3}}{2}$ or 4.33[01...] oe, soi	- Condone $\begin{pmatrix} 5 \\ 16.7 \text{ or } 16.66\dots \end{pmatrix}$ for 3 marks - Must be clearly identified as x and/or y or written in a vector; components may be e.g. embedded or doubled or implied by a clear diagram with either axes used OR north indicated or implied by the correct bearing - condone $i =$ and $j =$ for $x =$ and $y =$ For B2 (or B1 or SC1) accept e.g. the vectors (or the components $x = \dots$ and $y = \dots$) $-5\sqrt{3} \sin \frac{\pi}{3} \mathbf{i} + 5\sqrt{3} \sin \frac{\pi}{6} \mathbf{j}$ or $\begin{pmatrix} -5\sqrt{3} \sin 60 \\ 5\sqrt{3} \cos 60 \end{pmatrix}$ or $-5\sqrt{3} \cos \frac{\pi}{6} \mathbf{i} + 5\sqrt{3} \cos \frac{\pi}{3} \mathbf{j}$ or $\begin{pmatrix} -5\sqrt{3} \cos 30 \\ 5\sqrt{3} \sin 30 \end{pmatrix}$ or $\begin{pmatrix} 5\sqrt{3} \sin 300 \\ 5\sqrt{3} \cos 300 \end{pmatrix}$ oe - For B2 or B1, condone implication by e.g. $\begin{pmatrix} -7.5\mathbf{i} \\ 4.33\mathbf{j} \end{pmatrix}$

Question	Answer	Marks	Partial Marks	Guidance
7(d)	35.4[21...]	3	M2 for $[d^2=] (34-5)^2 + (37-(8+5\sqrt{3}))^2$ oe OR M1 for $34\mathbf{i} + 37\mathbf{j} - (5\mathbf{i} + (8+5\sqrt{3})\mathbf{j})$ or $29\mathbf{i} + (29-5\sqrt{3})\mathbf{j}$ oe M1 FT for $[d^2=]$ $(\text{their } 34 - \text{their } 5)^2 + (\text{their } 37 - (\text{their } 8 + \text{their } 5\sqrt{3}))^2$ oe FT, <i>their</i> $\overrightarrow{OA}$ and <i>their</i> $\overrightarrow{OB}$ providing both are of the form $a\mathbf{i} + b\mathbf{j}$ where a and b are non-zero constants	- Ignore incorrect labelling - Condone use of e.g. $\begin{pmatrix} 34\mathbf{i} \\ 37\mathbf{j} \end{pmatrix}$ For M1 FT: - If incorrect, the vector used for $\overrightarrow{OA}$ must be stated in this part. Allow use of $\overrightarrow{OA} = 18\mathbf{i} + 21\mathbf{j}$ for example, as long as they have stated it in this part. - If $\overrightarrow{OB}$ is incorrect in some way and is stated, please still allow the M1FT if possible, without checking back. If it is not stated, click back to (c) to check. Please allow the use of the correct $\overrightarrow{OB}$ or $\overrightarrow{OB} = 35\mathbf{i} + (8+5\sqrt{3})\mathbf{j}$ without them being explicitly stated. - Note: use of $35\mathbf{i} + (8+5\sqrt{3})\mathbf{j}$ and $34\mathbf{i} + 37\mathbf{j}$ leads to an answer of 20.3643.... and this can be given M1 FT even if it is all you see. - We must see EITHER a difference of vectors and the calculation for the square of the magnitude, or the magnitude, of the resultant vector OR the difference of components in a distance calculation as stated in the partial marks column.

Question	Answer	Marks	Partial Marks	Guidance
8(a)	$f(x) \geq 1.5$ oe	1		- Do not accept $x \geq 1.5$ - Accept e.g. $f \geq 1.5$ or $y \geq 1.5$ or $1 + 2^x \geq 1.5$ - Accept e.g. $f \in [1.5, \infty)$ or $f \in [1.5, \infty[$
8(b)(i)	$x \geq 1.5$ oe	1	FT $x \geq$ <i>their</i> 1.5 oe	- Must use x, do not accept e.g. $f(x) \geq 1.5$ - Accept e.g. $x \in [1.5, \infty)$ or $x \in [1.5, \infty[$

Question	Answer	Marks	Partial Marks	Guidance
8(b)(ii)	$\log_2(x-1)$ oe, nfw, mark final answer	2	M1 for full method for finding the inverse function: - resulting in a logarithmic expression - including swapping the variables and changing the subject	For M1 allow e.g. sign slips, arithmetic slips, omitted brackets and miscopies but not clear method errors such as an incorrect order of operations or an invalid application of logarithms
				M1 for e.g. - answer of $\log_2(x+1)$ (sign error) $2^y = x-1$ then $y = \log_2 x - 1$ (omitted brackets) M0 for e.g. $x = 1 + 2^y$ leading to $y = \log_2 x - 1$ (incorrect order of operations) $\log_2(y-1)$ (incomplete method) - use of $\lg x = \lg 1 + \lg 2^y$ (invalid application of logarithms) $x = 1 + 2^y$ leading to $y = 2^{x-1}$ (no logarithms used)
				- Any correct form is acceptable for 2 marks e.g. $\frac{\lg(x-1)}{\lg 2}$ or $\frac{\ln(x-1)}{\ln 2}$ - Accept $y = \dots$ or $f^{-1} =$ or even $f =$ but not $x =$

Question	Answer	Marks	Partial Marks	Guidance
8(b)(iii)	Correct smooth curves over the correct domain with intercepts correctly shown 	4	B1 for correct smooth curve for f in quadrants 1 and 2	- Ignore intercept and domain for this mark - If not over correct domain, then the correct shape includes tending to a horizontal asymptote in the second quadrant (may not be $y = 1.5$; allow tending to x-axis) - Ignore labels
			B1 for attempt at correct smooth curve drawn over correct domain	- No graph to the left of the point $(-1, 1.5)$ - Ignore intercept for this mark - If -1 is not marked on the x-axis, it should be roughly half the distance from the intersection of the inverse function to the origin, if this has been drawn
			B1 for $(0, 2)$ marked oe; must have attempted correct smooth curve	Allow this mark if $(2, 0)$ marked for the inverse function as long as there is no contradiction; may be stated on the side but must not just be in a list of points to plot
			B1 FT for the reflection of <i>their</i> f in the line $y = x$ maximum of 3 marks if not fully correct	- Must not have conflicting intercepts or other points marked if FT; the intercept 2 can be omitted for the graph of f^{-1} - They do not actually need to draw the line $y = x$ for the 4th mark
8(c)(i)	Valid explanation e.g. The range of f is subset of the domain of g .	1	dep on a correct answer to part 8(a) or $f \geq 1.5$ oe stated here	Accept statements that justify the range of f being a subset of the domain of g

Question	Answer	Marks	Partial Marks	Guidance
8(c)(ii)	$(1 + 2^x)(1 + 2^x + k)$ or $(1 + 2^x)^2 + (1 + 2^x)k$ or $1 + 2^x + k + 2^x + 2^{2x} + 2^x k$ oe, isw	1		- Allow recovery of missing brackets if seen - If brackets are missing in this part and recovered in 8(c)(iii), the mark in this part should not be awarded - Accept e.g. $1 + 2^{x+1} + k + (2^x)^2 + 2^x k$
8(c)(iii)	$k = 5$	2	M1 for $(1 + 2^3)(1 + 2^3 + k) = 126$ or $f(3) = 9$ and $9(9 + k) = 126$ oe	- Allow recovery of missing brackets for 2 marks; e.g. correct answer implies recovery - Not dep on and no FT from previous part

Question	Answer	Marks	Partial Marks	Guidance
9(a)	$v = 4e^{4t} - 20e^{2t}$	B1		- Accept e.g. $\frac{ds}{dt} = 4e^{4t} - 20e^{2t}$; - Condone e.g. $\frac{dy}{dx} = 4e^{4t} - 20e^{2t}$
	$\frac{dv}{dt} = 16e^{4t} - 40e^{2t}$	B1		May be e.g. $a = 16e^{4t} - 40e^{2t}$
	$\frac{dv}{dt} = 0$ and solves to find a value of e^{kt}	M1	FT <i>their</i> $\frac{dv}{dt}$ providing at least one term correct	
	$e^{2t} = \frac{\text{their}40}{\text{their}16}$ oe	M1	dep previous M1	Use of v rather than $\frac{dv}{dt}$ is M0
	$t = \frac{1}{2} \ln \frac{5}{2}$ isw or 0.458 or 0.4581[45...]	A1		
9(b)	96	2	M1 FT for $16e^{4\ln 2} - 40e^{2\ln 2}$ or for $\text{their} \frac{dv}{dt} \Big _{t=\ln 2}$, providing <i>their</i> $\frac{dv}{dt}$ is of the form $pe^{4t} + qe^{2t}$	If FT there must be evidence of the substitution of $t = \ln 2$ into <i>their</i> $\frac{dv}{dt}$

Question	Answer	Marks	Partial Marks	Guidance
10	Writes in terms of sine and cosine OR tangent and cosine e.g. $\frac{\sin(3x+1)}{\cos(3x+1)} = 5 \sin(3x+1)$ oe OR $\tan(3x+1) = 5 \tan(3x+1) \cos(3x+1)$	B1	OR Writes as a quadratic in $\cos^2(3x+1)$ oe: $25\cos^4(3x+1) - 26\cos^2(3x+1) + 1 = 0$	- May state and use a substitution e.g. $u = 3x + 1$ and $\frac{\sin u}{\cos u} = 5 \sin u$ - May state and use a substitution e.g. $y = \cos^2(3x + 1)$ and $25y^2 - 26y + 1 = 0$ $\sin(3x + 1) - 5 \sin(3x + 1)$ $\cos(3x + 1) = 0$ - B0 for e.g. $\frac{\sin}{\cos}(3x + 1) = 5 \sin(3x + 1)$ unless recovered before e.g. any incorrect cancelling of $\sin(3x + 1)$; - May imply B1 by e.g. $\sin(3x + 1) = 0$ and $\cos(3x + 1) = \frac{1}{5}$
	$\sin(3x + 1) = 0$ nfw OR $\tan(3x + 1) = 0$	B1	OR $[(25\cos^2(3x + 1) - 1)(\cos^2(3x + 1) - 1)]$ $\cos(3x + 1) = [\pm] 1$ nfw	- Accept from e.g. $(25y - 1)(y - 1)$ but must be stated as values of $\cos(3x + 1)$ not y. - e.g. B0 B1 B0 M1 A1 is possible
	$\cos(3x + 1) = \frac{1}{5}$ oe, nfw	B1	OR $[(25\cos^2(3x + 1) - 1)(\cos^2(3x + 1) - 1)]$ $\cos(3x + 1) = [\pm] \frac{1}{5}$ nfw	- Can be awarded following $\frac{\cancel{\sin}}{\cos}(3x + 1) \times \frac{1}{5 \cancel{\sin(3x + 1)}} = 1$ (do not consider this incorrect form to be XP) - e.g. B0 B0 B1 M1 A1 is possible

Question	Answer	Marks	Partial Marks	Guidance
10	Finds at least one value for $3x + 1$: $3x + 1 = 0$, π nfw $3x + 1 = \pm 1.3694\dots$,	M1		- Accept prematurely rounded values e.g. 3.1 for π or ± 1.36 or ± 1.37 or ± 1.4 for $\pm 1.369\dots$ - Candidates working in degrees do not earn any method marks until they convert their values to radians. - Implied by one correct value of x
	$-\frac{1}{3}$ or $-0.333[3\dots]$ 0.714 or 0.7138[6...] 0.123 or 0.1231[4...] $-0.79[0]$ or $-0.7898[1\dots]$	A2	and no extras in $-\frac{\pi}{3} < x < \frac{\pi}{3}$ A1 for any two correct angles, ignoring extras	- Answers outside $-1.04719\dots < x < 1.04719\dots$ can be ignored even if they are incorrect - Accept $\frac{\pi-1}{3}$ for 0.714 - Mark the most accurate version of each angle seen and ignore any subsequent incorrect rounding - If a value is rejected, it cannot be credited -0.789 is not sufficiently accurate to count as it is truncated to 3 sf - Ignore any incorrect units, if stated

Question	Answer	Marks	Partial Marks	Guidance
11	EITHER Solution 1 Forms the equation of C_1 : $(x-3)^2 + (y-5)^2 = 5$ oe, soi or $x^2 + y^2 - 6x - 10y + 29 = 0$ oe, soi	(B2)	B1 for $(x-3)^2 + (y-5)^2 = k$, where k is a constant soi or for $r^2 = 5$ or $r = \sqrt{5}$ oe, soi or for $x^2 + y^2 - 6x - 10y + c = 0$ or for $x^2 + y^2 + ax + by + 29 = 0$	- Any correct equation for B2 - May be unsimplified for B2 or B1 - Allow B1 for e.g. $(1-3)^2 + (4-5)^2 = 5$
	Uses $y = 19 - 3x$ and <i>their</i> C_1 to correctly eliminate one unknown e.g. $(x-3)^2 + (19-3x-5)^2 = \text{their } 5$	M1	FT <i>their</i> C_1 providing of the form $(x-3)^2 + (y-5)^2 = k$ or $x^2 + y^2 - 6x - 10y + c = 0$	
	Correct unsimplified equation e.g. $(x-3)^2 + (19-3x-5)^2 = 5$	A1		
	Writes as a correct 3-term quadratic equation in solvable form e.g. $10x^2 - 90x + 200 [= 0]$	A1		
	Solves or factorises <i>their</i> 3-term quadratic	M1	dep previous M1	Attempt to solve <i>their</i> quadratic in form $ax^2 + bx + c = 0$ by: - factorising – factorisation must result in correct a and c for FT equation, or scaled form of FT equation. - quadratic formula – discriminant of FT equation must not be negative. Formula must be correct but condone one sign error in substitution. - completing the square – must get as far as $(x + \frac{b}{2})^2 = -\frac{c}{a} + \frac{b^2}{4}$ or equivalent, condone one sign error.

Question	Answer	Marks	Partial Marks	Guidance
11	Finds the correct pair of values for one unknown e.g. $x = 4$, $x = 5$	A1		
	Finds at least one correct point of intersection of the two circles (4, 7) or (5, 4) or states $(x-12)^2 + (19-3x-8)^2 = r^2$ oe	A1		- Point of intersection can be implied by e.g. $\sqrt{(12-5)^2 + (8-4)^2}$ or a correct answer if not from wrong working $10x^2 - 90x + 265 = r^2$ oe
	$\sqrt{65}$ oe, nfw	A1)	dep on all previous marks awarded	- Ignore attempts to decimalise $\sqrt{65}$ or change form - Condone 8.06[225...] - A0 for final answer $\pm \sqrt{65}$
	OR Solution 2 Forms the equation of C_1 : $(x-3)^2 + (y-5)^2 = 5$ oe, soi	(B2	Compares equations of the common chord B1 for $(x-3)^2 + (y-5)^2 = k$, where k is a constant soi or for $r^2 = 5$ or $r = \sqrt{5}$ oe, soi or for $x^2 + y^2 - 6x - 10y + c = 0$ or for $x^2 + y^2 + ax + by + 29 = 0$	- Any correct equation for B2 - May be unsimplified for B2 or B1 - Allow B1 for e.g. $(1-3)^2 + (4-5)^2 = 5$ - Implied by correct expanded form
	Forms the equation of C_2 : $(x-12)^2 + (y-8)^2 = r^2$ oe, soi	B1		- Any correct equation - Implied by correct expanded form
	$x^2 + y^2 - 6x - 10y + 34 - their 5 = 0$ and $x^2 + y^2 - 24x - 16y + 208 - r^2 = 0$	M1	FT <i>their</i> C_1 providing of the form $(x-3)^2 + (y-5)^2 = k$	- Expands and simplifies both equations - Does not have to be equal to 0 but must have collected any like terms, including constants

Question	Answer	Marks	Partial Marks	Guidance
11	Subtracts $x^2 + y^2 - 6x - 10y + 29 = 0$ and $x^2 + y^2 - 24x - 16y + 208 - r^2 = 0$ to find $-6x + 24x - 10y + 16y + 29 - 208 + r^2 = 0$ oe	A1		- Subtracts two correct expanded circle equations to eliminate x^2 and y^2 - Must be a correct equation, not an expression
	$18x + 6y - 179 + r^2 = 0$ oe	A1		Finds simplified equation of common chord
	Eliminating x and y using e.g. $y = 19 - 3x$ and $18x + 6y - 179 + r^2 = 0$	M1	dep previous M1 FT $18x + 6y + (their - 179) + r^2 = 0$	$y + 3x = 19$ and $y + 3x = \frac{their\ 179 - r^2}{6}$
	$19 = \frac{179 - r^2}{6}$ oe	A1		Eliminates x and y
	$\sqrt{65}$ oe, nfw	A1)	dep on all previous marks awarded	- Ignore attempts to decimalise $\sqrt{65}$ or change form - Condone 8.06[225...] - A0 for final answer $\pm \sqrt{65}$
	OR Solution 3 Radius of C_1 : $r = \sqrt{5}$	(B1	Repeated Pythagoras using the line joining the centres	
	Equation of the line joining the centres e.g. $3y - x = 12$ or $y - 5 = \frac{1}{3}(x - 3)$ oe	B1		Any correct form e.g. $y = \frac{x}{3} + 4$ or $y - 8 = \frac{1}{3}(x - 12)$
	Uses $y = 19 - 3x$ and <i>their</i> $3y - x = 12$ to correctly eliminate one unknown	M1		
	Finds the midpoint of the common chord: (4.5, 5.5)	A1		

Question	Answer	Marks	Partial Marks	Guidance
11	Distance calculation using <i>their</i> (4.5, 5.5) and (3, 5) or (12, 8) e.g. $\sqrt{((their4.5)-3)^2 + ((their5.5)-5)^2}$	M1	dep previous M1	If FT evidence of substitution into the distance formula/ Pythagoras must be seen
	Distance from (4.5, 5.5) to (3, 5) : $\frac{\sqrt{10}}{2}$ isw, or 1.58[11...] oe Distance from (4.5, 5.5) to (12, 8): $\frac{5\sqrt{10}}{2}$ isw or 7.91 or 7.905[69...] oe	A2	A1 for each	
	Half the length of the common chord: $\frac{\sqrt{10}}{2}$ isw, or 1.58[11...] oe	A1		
	$\sqrt{65}$ oe, nfw	A1)	dep on all previous marks awarded	- Must be using half the length of the common chord not the distance from (4.5, 5.5) to (3, 5) - Ignore attempts to decimalise $\sqrt{65}$ or change form - Condone 8.06[225...] - A0 for final answer $\pm \sqrt{65}$
	OR Solution 4 Forms the equation of C_1 : $(x-3)^2 + (y-5)^2 = 5$ oe, soi or $x^2 + y^2 - 6x - 10y + 29 = 0$ oe, soi	(B2)	Solving simultaneous circle equations B1 for $(x-3)^2 + (y-5)^2 = k$, where k is a constant soi or for $r^2 = 5$ or $r = \sqrt{5}$ oe, soi or for $x^2 + y^2 - 6x - 10y + c = 0$ or for $x^2 + y^2 + ax + by + 29 = 0$	- Starts as the main method but does not solve to find values of x - Any correct equation for B2 - May be unsimplified for B2 or B1 - Allow B1 for e.g. $(1-3)^2 + (4-5)^2 = 5$

Question	Answer	Marks	Partial Marks	Guidance
11	Uses $y = 19 - 3x$ and <i>their</i> C_1 to correctly eliminate one unknown e.g. $(x - 3)^2 + (19 - 3x - 5)^2 = \text{their } 5$	M1	FT <i>their</i> C_1 providing of the form $(x - 3)^2 + (y - 5)^2 = k$ or $x^2 + y^2 - 6x - 10y + c = 0$	
	Correct unsimplified equation e.g. $(x - 3)^2 + (19 - 3x - 5)^2 = 5$	A1		
	Simplifies e.g. $10x^2 - 90x = -200$	A1		
	Forms the equation of C_2 : $(x - 12)^2 + (y - 8)^2 = r^2$ oe, soi	B1		- Any correct equation - Implied by correct expanded form
	Uses $y = 19 - 3x$ and <i>their</i> C_2 to correctly eliminate one unknown e.g. $(x - 12)^2 + (19 - 3x - 8)^2 = r^2$	M1	dep previous M1	
	Correct simplified equation e.g. $10x^2 - 90x + 265 = r^2$	A1		
	$\sqrt{65}$ oe, nfw	A1)	dep on all previous marks awarded	- Solves $10x^2 - 90x + 200 = 0$ and $10x^2 - 90x + 265 = r^2$ - Ignore attempts to decimalise $\sqrt{65}$ or change form - Condone 8.06[225...] - A0 for final answer $\pm \sqrt{65}$

Question	Answer	Marks	Partial Marks	Guidance
12	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or $\frac{dV}{dr} = 9$ so i	M1	implied by e.g. <i>their</i> $4\pi r^2 = 9$ providing it is not from wrong working	- Allow for any relevant chain rule stated e.g. $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ - Implied by $\frac{dV}{dt} = \left(\text{their} \frac{dV}{dr} \right) \times \frac{dr}{dt}$ but not implied by $\frac{dV}{dt} = 9 \times \frac{dr}{dt}$ which is given
	$4\pi r^2 = 9$	M1	dep previous M1	- Allow for e.g. $4\pi r^2 \times \frac{dr}{dt} = 9 \times \frac{dr}{dt}$ - BOD M1 for $\frac{dV}{dr} = 4\pi r^2$ and $4\pi r^3 = 9r$ - May be from e.g. $\frac{dV}{dr} \rightarrow \frac{dV}{dt} \cdot \frac{dt}{dr} = 9 \times \frac{dr}{dt} \cdot \frac{dt}{dr} = 4\pi r^2$ or $\frac{dV}{dt} \cdot \frac{dt}{dV} = 9 \times \frac{dr}{dt} \cdot \frac{dt}{dV} \rightarrow 1 = 9 \times \frac{dr}{dV}$ $\rightarrow 1 = 9 \times \frac{1}{4\pi r^2}$

Question	Answer	Marks	Partial Marks	Guidance
12	$V = \frac{9}{2\sqrt{\pi}}$ oe, isw	A2	A1 for $r = \sqrt{\frac{9}{4\pi}}$ or $\frac{3}{2\sqrt{\pi}}$ oe, isw	• $V \left[= \frac{4}{3} \pi \left(\frac{3}{2\sqrt{\pi}} \right)^3 \right] = 2.538853... \rightarrow$ accept 2.54 or awrt 2.54 or e.g. $\frac{9\pi}{2\pi\sqrt{\pi}}$ • $\frac{4}{3} \pi \left(\frac{3}{2\sqrt{\pi}} \right)^3$ alone is not sufficient for A2 but implies A1 • $r = 0.8462843...$ rot to 2 or more sf • Condone $r = 0.84$ or 0.85 oe as they will lose the last mark Pre unless they recover • Must state or use positive value of r only