

CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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4037/13

October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

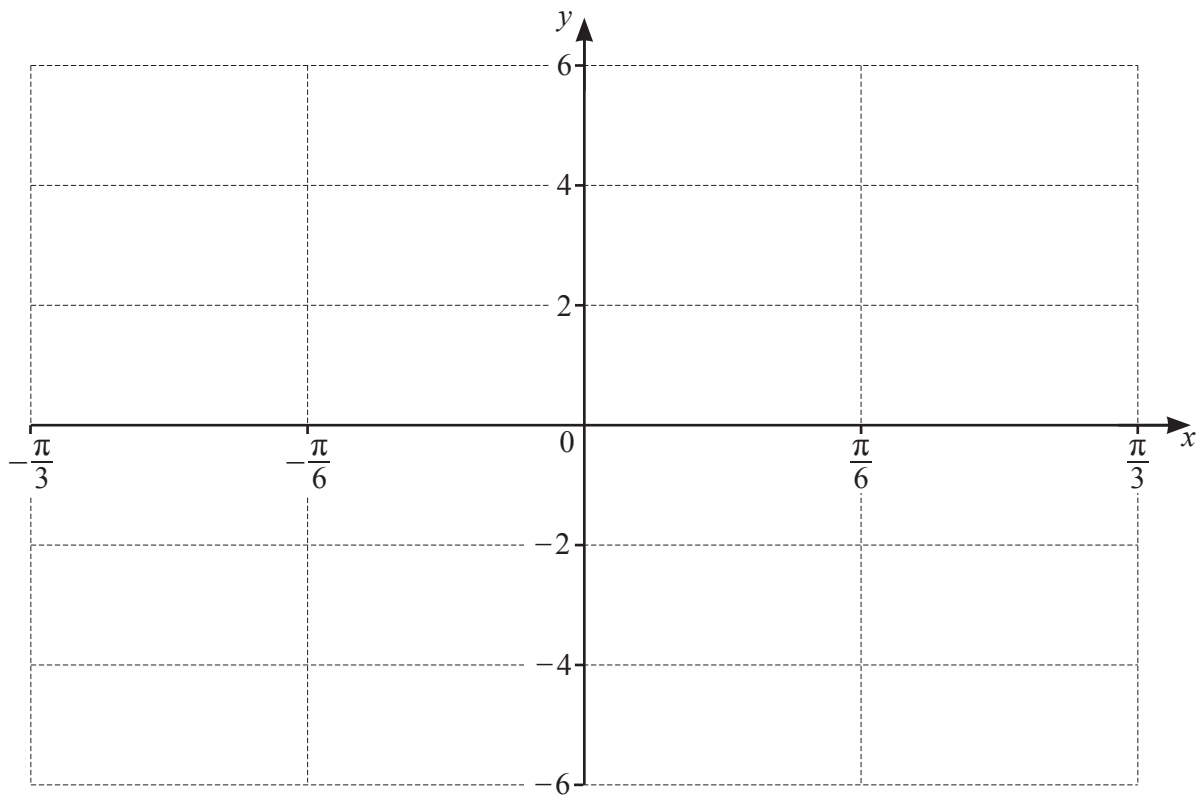
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 On the axes, sketch the graph of $y = 4 \sin 3x - 2$ for $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

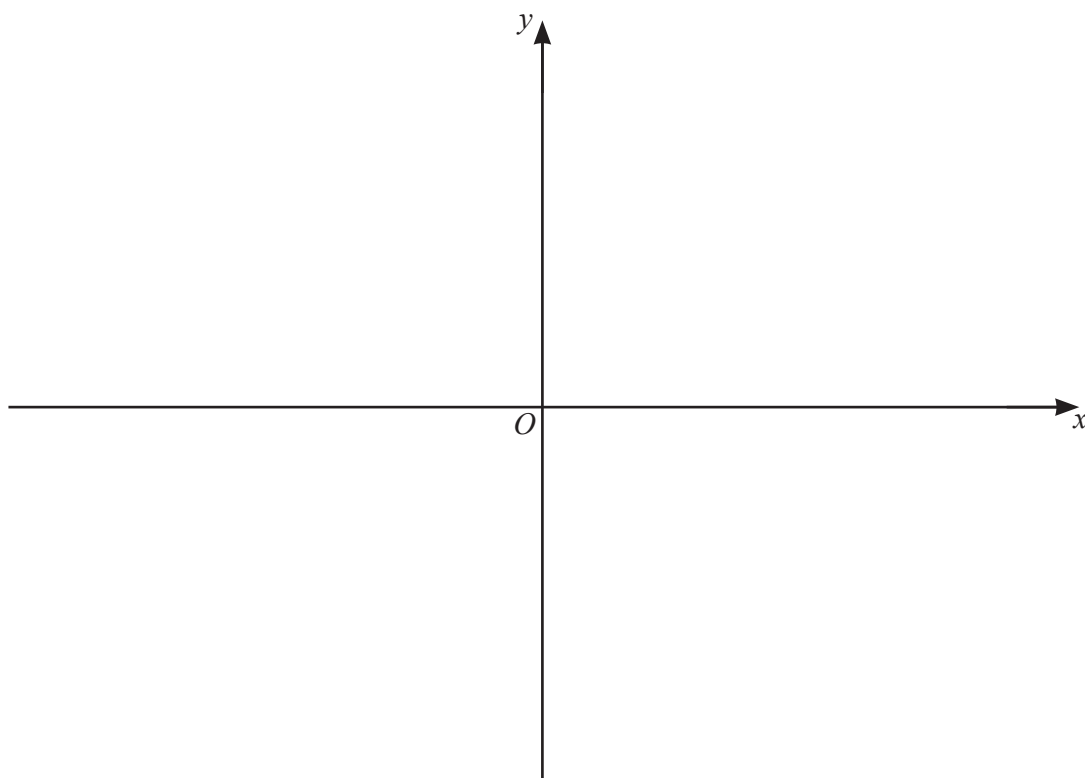
[3]



- 2 (a) Show that $2x^2 + x - 15$ can be written in the form $2(x+a)^2 + b$, where a and b are exact constants to be found. [2]

- (b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$. [2]

- (c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



- (d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions. [1]

- 3 (a) Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$

[3]

- (b) Solve the equation $\log_3 x + 3 = 10 \log_x 3$, giving your answers as powers of 3.

[4]

- 4 The polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a , b and c are integers. It is given that $p'(0) = -9$.

(a) Show that $b = -9$. [1]

It is also given that $3x + 2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c . [4]

(c) Find the quadratic $q(x)$ such that $p(x) = (3x + 2) \times q(x)$. [1]

(d) Hence find $p(x)$ as a product of linear factors with integer coefficients. [1]

- 5** A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

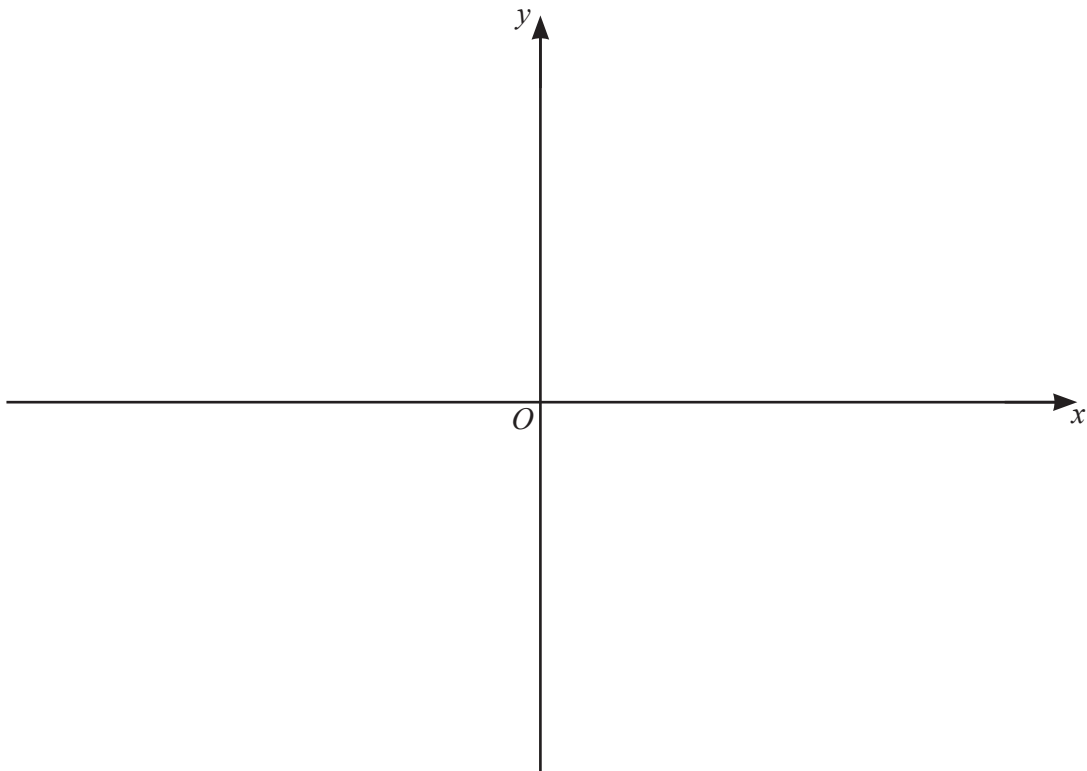
(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

6 A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f . [1]

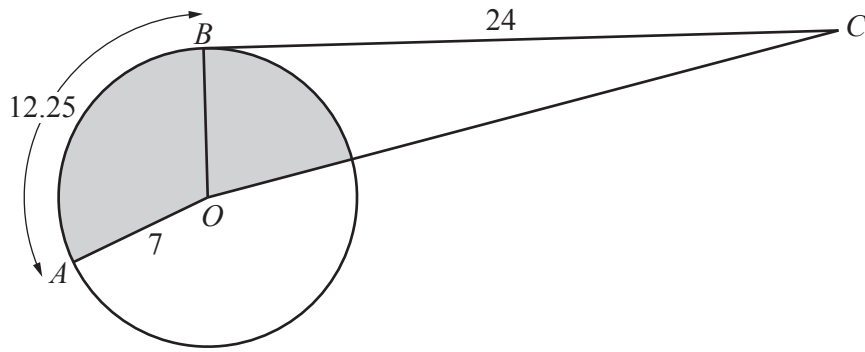
(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



- 7 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4 \sin 2x + 1) dx$. [5]

8 In this question all lengths are in metres.



The diagram shows a circle, centre O , radius 7 . The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24 . The length of the minor arc AB is 12.25 .

(a) Find the obtuse angle AOB , giving your answer in radians. [1]

(b) Find the perimeter of the shaded region. [4]

(c) Find the area of the shaded region.

[2]

9 A 6-character password is to be formed from the following characters.

Letters	A	B	C	D
Numbers	1	2	3	4
Symbols	*	#	\$	£

No character may be used more than once in any password.

(a) (i) Find the number of different 6-character passwords that can be formed.

[1]

(ii) How many of these 6-character passwords end with a symbol?

[1]

(b) Find the number of different 6-character passwords that include all the symbols, but do not start or end with a symbol.

[2]

- 10** Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where x is in radians, for $-1.5 \leq x \leq 1.5$. [5]

- 11 It is given that $\int_1^a \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x} \right) dx = \ln \frac{1}{5}$, where $a > 1$. Find the exact value of a . [6]

12 It is given that $y = \frac{(3x^2 - 2)^{\frac{2}{3}}}{x - 1}$, for $x > 1$.

(a) Write $\frac{dy}{dx}$ in the form $\frac{(3x^2 - 2)^{-\frac{1}{3}}}{(x - 1)^2}(x^2 + Ax + B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to $2 + p$, where p is small. [2]

- 13** The points P and Q have coordinates $(5, -12)$ and $(15, -6)$ respectively. The point R lies on the line l , the perpendicular bisector of the line PQ . The x -coordinate of R is 7.

(a) Find the y -coordinate of R .

[4]

(b) The point S lies on l such that its distance from PQ is 3 times the distance of R from PQ . Find the coordinates of the two possible positions of S .

[3]

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