



# Cambridge O Level

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**ADDITIONAL MATHEMATICS**
**4037/12**

Paper 1

**October/November 2024**
**2 hours**

You must answer on the question paper.

No additional materials are needed.

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

 This document has **16** pages. Any blank pages are indicated.



## Mathematical Formulae

### 1. ALGEBRA

#### Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*  $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*  $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

### 2. TRIGONOMETRY

#### Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

#### Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$





- 1 The curve  $y = a \cos bx + c$ , where  $a$ ,  $b$  and  $c$  are integers, passes through the points  $\left(-\frac{\pi}{6}, -2\right)$  and  $\left(\frac{\pi}{9}, \frac{1}{2}\right)$ . The curve has a period of  $\frac{2\pi}{3}$ .
- (a) Find the values of  $a$ ,  $b$  and  $c$ . [4]

- (b) Find the least value of  $y$  on the curve for  $0 \leq x \leq \frac{\pi}{2}$ , and state the value of  $x$  at which this occurs. [3]





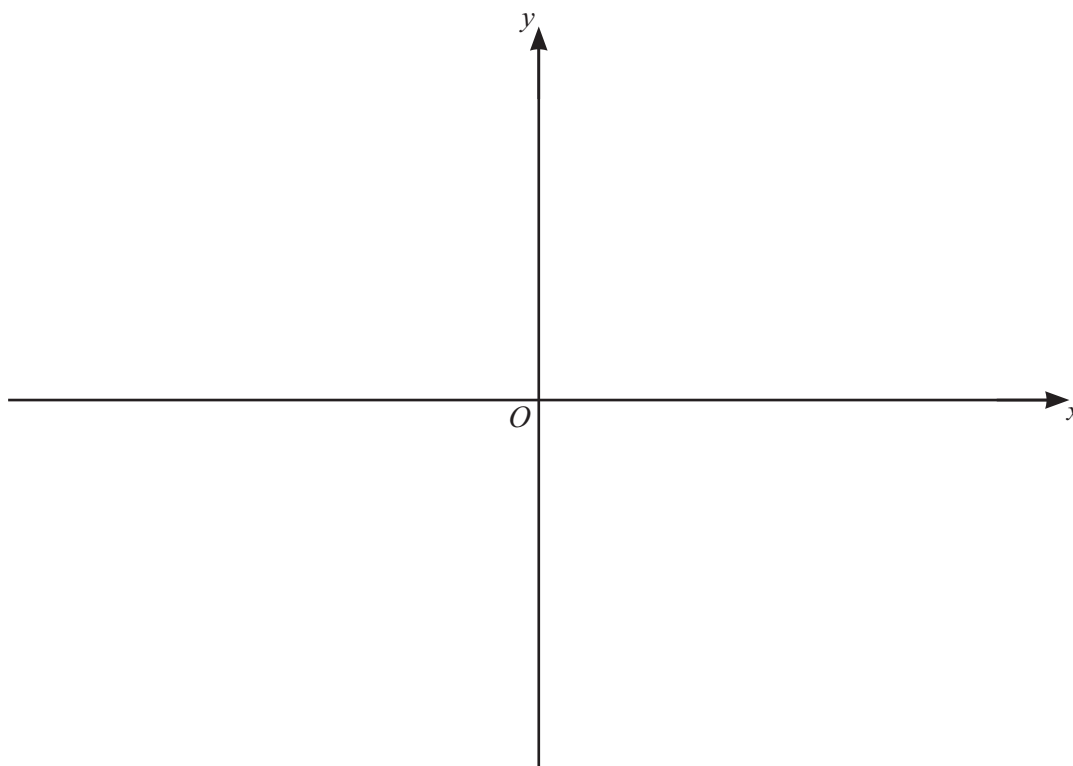
2 It is given that  $y = f(x)$ , where  $f(x) = (2x - 5)(x - 1)^2$ .

(a) Find the coordinates of the stationary points on the curve  $y = f(x)$ .

[4]

(b) On the axes, sketch the graph of  $y = f(x)$ , stating the intercepts with the axes.

[3]

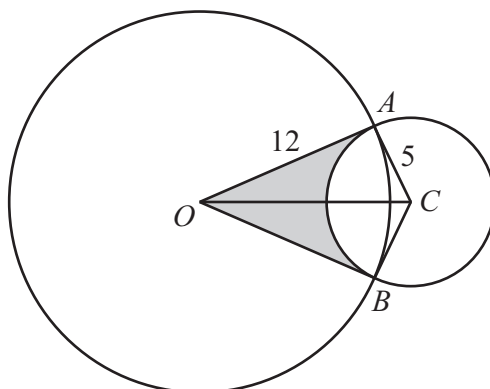


(c) Hence find the values of  $k$  for which  $f(x) = k$  has exactly one solution.

[2]



- 3 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre  $O$  and radius 12, and a circle with centre  $C$  and radius 5. The circles intersect at the points  $A$  and  $B$ , such that  $OA$  and  $OB$  are tangents to the circle with centre  $C$ .

- (a) Show that the obtuse angle  $ACB$  is 2.35 radians, correct to 2 decimal places. [2]

- (b) Find the perimeter of the shaded region. [2]

- (c) Find the area of the shaded region. [3]





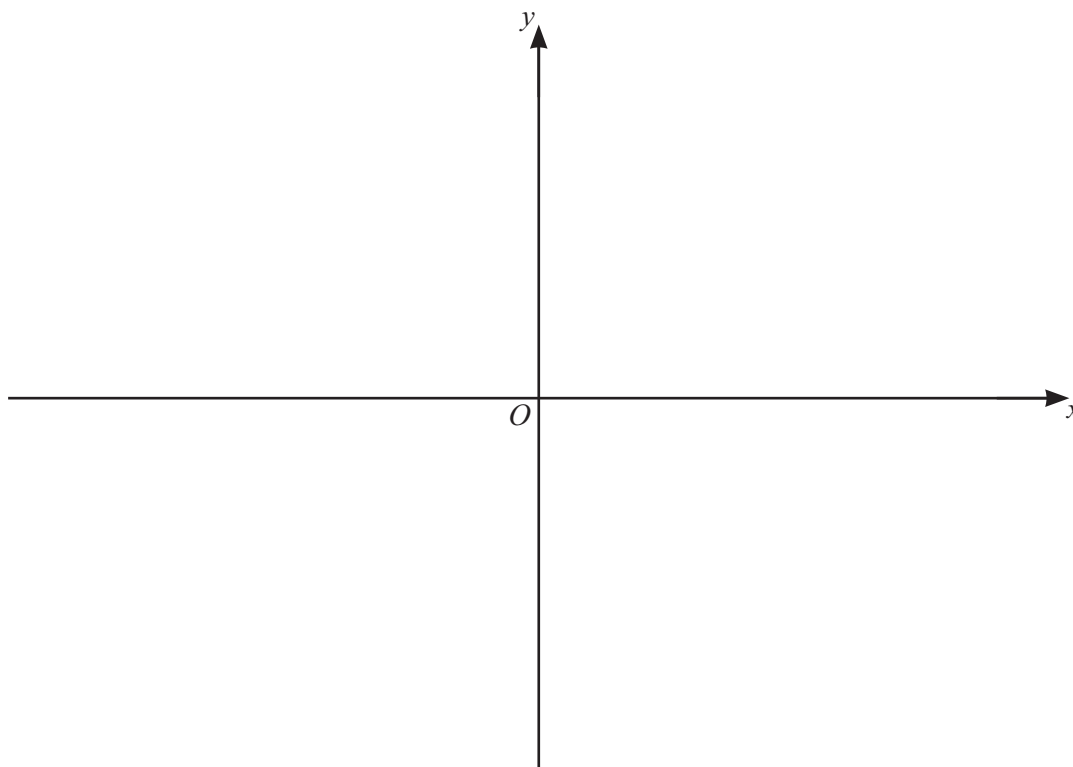
4 The function  $f$  is such that  $f(x) = 4 \ln(3x - 2)$ , for  $x > a$ , where  $a$  is as small as possible.

(a) (i) Write down the value of  $a$ . [1]

(ii) Write down the range of  $f$ . [1]

(iii) Find  $f^{-1}(x)$ , stating its domain and range. [4]

(iv) On the axes sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$ , stating the intercepts with the axes. [4]





(b) Given that  $g(x) = (2x + 1)^{\frac{1}{2}} + 4$ , for  $x > 0$ , solve the equation  $gg(x) = 9$ .

[3]





5 (a) Show that  $\frac{1 + \cot^2 \theta}{\cot^2 \theta} = \sec^2 \theta$ .

[1]

(b) Write down the derivative of  $\tan \theta$  with respect to  $\theta$ .

[1]

(c) Using **part (a)** and **part (b)**, find the exact value of  $\int_0^{\frac{\pi}{3}} \left( \frac{1 + \cot^2 \theta}{\cot^2 \theta} - \sin \theta \right) d\theta$ .

[4]





- 6 (a) Find, in descending powers of  $x$ , the first 3 terms in the expansion of  $\left(x + \frac{2}{x^2}\right)^{10}$ . Simplify each term as far as possible. [3]

- (b) Find the term independent of  $x$  in the expansion of  $\left(4x^2 + \frac{1}{2x^2}\right)^8$ . [2]





- 7 It is given that  $y = \frac{\ln(3x^2 - 1)}{x + 2}$ , for  $x > \frac{1}{\sqrt{3}}$ . When  $x = 1$ ,  $y$  is increasing at the rate of  $h$  units per second. Find, in terms of  $h$ , the corresponding rate of change in  $x$ , giving your answer in exact form.

[6]

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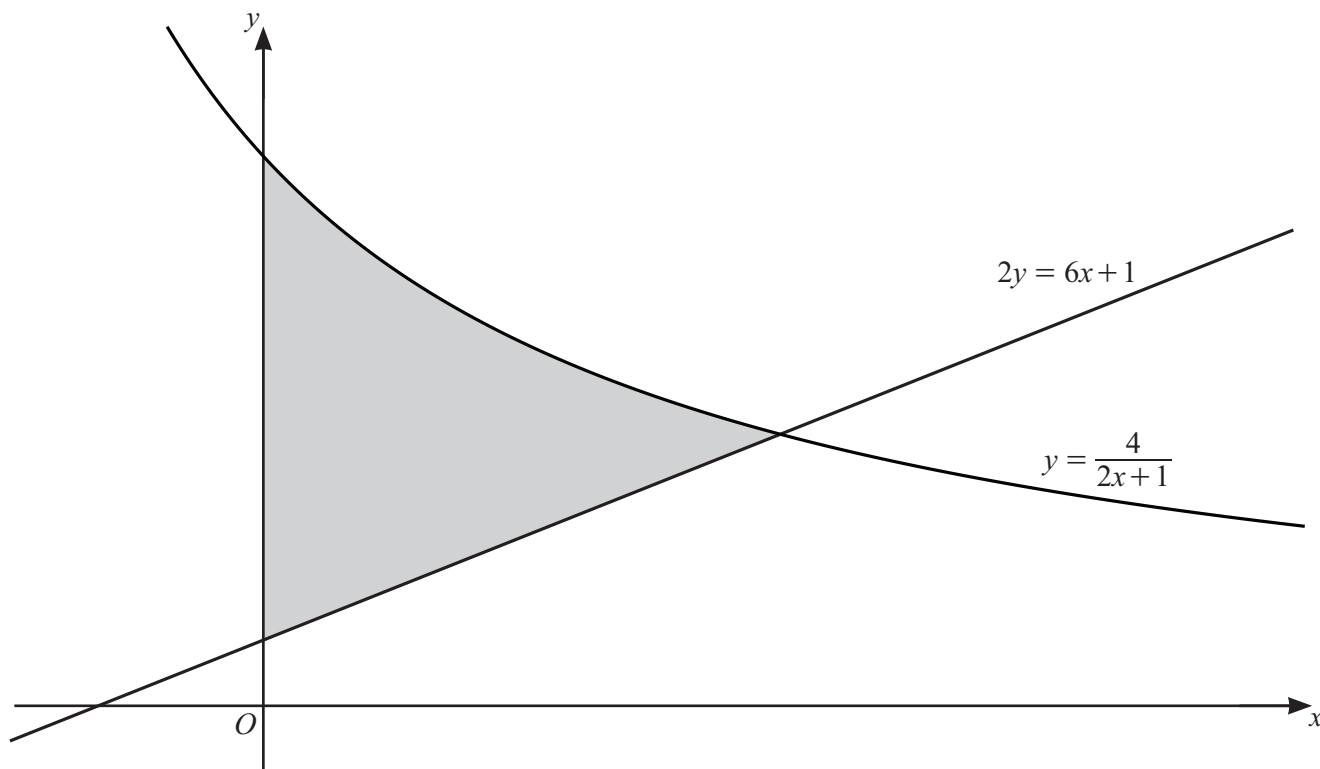


- 8 The tangent to the curve  $y = e^x(2x+5)^{\frac{1}{2}}$  at the point where  $x = 2$  meets the  $x$ -axis at the point  $X$  and the  $y$ -axis at the point  $Y$ . Find the coordinates of the mid-point of  $XY$ , giving your answer in exact form. [8]





9



The diagram shows part of the curve  $y = \frac{4}{2x+1}$  and the straight line  $2y = 6x + 1$ . Find the area of the shaded region, giving your answer in the form  $\ln a + b$ , where  $a$  is an integer and  $b$  is a rational number. [8]





Continuation of working space for question 9.





- 10 (a) The first 3 terms of an arithmetic progression are  $2 \tan 2x$ ,  $5 \tan 2x$ ,  $8 \tan 2x$ . Find the values of  $x$ , where  $-180^\circ \leq x \leq 180^\circ$ , for which the sum to 30 terms is  $455\sqrt{3}$ . [5]

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(b) The first 3 terms of a geometric progression are

$$5 \cos^2\left(\theta - \frac{\pi}{2}\right), \quad 20 \cos^4\left(\theta - \frac{\pi}{2}\right), \quad 80 \cos^6\left(\theta - \frac{\pi}{2}\right), \quad \text{where } -\frac{\pi}{6} \leq \theta \leq \frac{7\pi}{6}.$$

Find the values of  $\theta$  for which this geometric progression has a sum to infinity.

[6]





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