

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9231 FURTHER MATHEMATICS

9231/21

Paper 2, maximum raw mark 100

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Page 2	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

Question Number	Mark Scheme Details		Part Mark	Total
1	Equate impulse to momentum to find initial speed v and Newton's law of restitution to find new speed:	$v = 4u, v' = ev = [-] 3u$ M1 A1	2	2
2	Find v^2 at both A and B : Find amplitude a m from given K.E. ratio: Find ω from $v_{\max} = a\omega$: Find time ($\sqrt{}$ on a) at A or at B , e.g.: Combine correctly to find time from A to B : Evaluate to 3 d.p.:	$v_A^2 = \omega^2(a^2 - 0.5^2)$ and $v_B^2 = \omega^2(a^2 - 0.75^2)$ B1 $\frac{1}{2}mv_A^2 = (12/11) \frac{1}{2}mv_B^2$ $11(a^2 - 0.5^2) = 12(a^2 - 0.75^2)$ $a^2 = \frac{1}{4}(27 - 11) = 4, a = 2$ M1 A1 $0.6 = 2\omega, \omega = 0.3$ B1 $\omega^{-1} \sin^{-1}(0.5/2)$ or $\omega^{-1} \cos^{-1}(0.5/2)$ $\omega^{-1} \sin^{-1}(0.75/2)$ or $\omega^{-1} \cos^{-1}(0.75/2)$ M1 A1 $\omega^{-1} \sin^{-1}(0.75/2) - \omega^{-1} \sin^{-1}(0.5/2)$ or $\omega^{-1} \cos^{-1}(0.5/2) - \omega^{-1} \cos^{-1}(0.75/2)$ M1 $= \omega^{-1}(0.3844 - 0.2527)$ or $\omega^{-1}(1.318 - 1.186)$ $= 1.2813 - 0.8423$ $4.3937 - 3.9547 = 0.439$ [s] A1	3	8

Page 3	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$	M1	7	10
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{2}mu^2$	M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10, v_B = (1 + e)u/10$			
		or $v_A, v_B = -u/2, u/6$ [or $7u/10, u/30$]	M1 A1		
	Use in 3rd eqn to find e , e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$			
	(A0 if finally $\pm \frac{2}{3}$)	$90e^2 = 40, e = \frac{2}{3}$	M1 A1		
	Use Newton's law of restitution with	$v_C = 2v_B', \text{ e.g.: } v_C - v_B' = ev_B, v_B' = \frac{2}{3}v_B$	B1	3	10
		$[v_B = u/6, v_B = u/9, v_C = 2u/9]$			
	Use conservation of momentum to find k :	$9mv_B' + kmv_C = 9mv_B$			
		$9v_B' + 2kv_B' = 13.5v_B', k = 9/4$	M1 A1		

Page 4	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

4	(i)	Use conservation of energy at lowest point: Use $F = ma$ radially at lowest point: Eliminate v^2 to find R [$v^2 = 2.3 ga$]:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$ $R - mg = mv^2/a$ $R = mu^2/a + 3mg = 3.3mg$	B1 B1 B1	3	10
	(ii)	Use conservation of energy at B to find V_B : (A.E.F.)	$\frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga \sin \theta$ $V_B^2 = (0.3 + 0.5)ga, V_B = \sqrt{(0.8ga)}$ or $2\sqrt{(ga/5)}$ or $0.894\sqrt{(ga)}$	M1A1 A1		
	(iii)	Use vertical component v_B of speed V_B at B : Find height h reached above B : Find height h reached above level of O :	$v_B = V_B \cos \theta [= \frac{1}{4}\sqrt{15} V_B = \sqrt{(3/4 ga)}]$ $h = v_B^2/2g = 3a/8$ $h - a \sin \theta = 3a/8 - \frac{1}{4}a = a/8$ A.G.	M1 M1 A1 A1		
5		Find MI of components about A : (M1 for BC or CD)	Glass $\{ \frac{1}{3}(5a)^2 + 25a^2 \} = 20Ma^2$ $AB \quad M\{ \frac{1}{3}(4a)^2 + (4a)^2 \} = 64Ma^2/3$ $AD \quad \frac{1}{3}M\{ \frac{1}{3}(3a)^2 + (3a)^2 \} = 4Ma^2$ $BC \quad \frac{1}{3}M\{ \frac{1}{3}(3a)^2 + 73a^2 \} = 76Ma^{2/3}$ $CD \quad M\{ \frac{1}{3}(4a)^2 + 52a^2 \} = 172Ma^{2/3}$	M1 A1 B1 B1 M1 A1 A1	8	13
		Find total MI about A : (OR can first find total MI about centre of mass) State or imply total mass acts at mid-point of AC	$I = 128Ma^2$ A.G.	A1 M1		
		Use eqn of circular motion to find $d^2\theta/dt^2$: Approximate $\sin \theta$ by θ and substitute for I : Find period $T = 2\pi/\omega$ with $\omega = \sqrt{(49g/384a)}$:	$I d^2\theta/dt^2 = [-] (49Mg/15) 5a \sin \theta$ $d^2\theta/dt^2 = -(49g/384a) \theta$ $T = 2\pi\sqrt{(384a/49g)}$ or $(16\pi/7)\sqrt{(6a/g)}$ or $17.6\sqrt{(a/g)}$ (A.E.F.)	M1 A1 A1 B1		

Page 5	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

6		State or find the expected value of X : using $p = \frac{1}{4}$:	$E(X) = 1/p = 1/\frac{1}{4} = 4$	B1	1	5
	(i)	Find $P(X = 4)$:	$P(X = 4) = (\frac{3}{4})^3 \frac{1}{4} = 27/256$ or 0.105	M1 A1	2	
	(ii)	Find $P(X < 6)$:	$P(X < 6) = 1 - (\frac{3}{4})^5$ or $\{1 + \frac{3}{4} + (\frac{3}{4})^2 + (\frac{3}{4})^3 + (\frac{3}{4})^4\} \frac{1}{4}$ $= 781/1024$ or 0.763	M1 A1	2	
		S.R. Using $p = \frac{1}{2}$ can earn B0 M1 A0 M0 A0				
7	(i)	State probability density function of T :	$f(t) = 0.001 \exp(-0.001 t) \quad (t \geq 0)$ [$= 0$ (otherwise or $t < 0$)]	B1	1	8
	(ii)	Find $P(T > 2000)$: S.R. $1 - e^{-2} = 0.865$ earns B1 only (max 1/3) State inequality for t (lose A1 if $=$ or $\leq$): Solve for $t_{\max}$: (Omitting power 10 earns 0/4; using $1 - (\exp(-0.001 t))^{10}$ can earn M1 A0 M1 A0 only)	$P(t > 2000) = 1 - F(2000)$ $= 1 - (1 - e^{-2}) = e^{-2}$ or 0.135 $(\exp(-0.001 t))^{10} \geq [or >] 0.9$ $t_{\max} = (\ln 0.9) / (-0.01) = 10.5$	M1 M1 A1 M1 A1 M1 A1	3 4	

Page 7	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

9	Find expected frequency p :	$p = 200 \int_2^3 (1/x \ln 8) dx$ $= (200 / \ln 8) [\ln x]_2^3$ $= 200 \times 0.1950 = 39.00$ A.G. $q = 21.46$ <i>or</i> 21.45	M1A1 M1A1	4	
	Find q by similar method <i>or</i> by using total of 200:				
10	State (at least) null hypothesis:	$H_0: f(x)$ fits data (A.E.F.)	B1	6	10
	Calculate χ^2 (to 3 s.f.):	$\chi^2 = 0.202 + 0.923 + 0.678 + 0.584$ $+ 1.134 + 4.134 + 3.644 = 11.3$	M1A1		
	State or use correct tabular χ^2 value (to 3 s.f.):	$\chi_{6, 0.95}^2 = 12.59$	B1		
	Valid method for reaching conclusion:	Accept H_0 if $\chi^2 \leq$ tabular value	M1		
	Conclusion consistent with correct values (A.E.F):	Distribution fits observations	A1		
10	Find correlation coefficient r :			4	
	$r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)(55\,991 - 639^2 / 10)\}}$ (A.E.F.; A0 if only 3 s.f. clearly used)	$= 18\,189.6 / \sqrt{(46\,280.4 \times 15\,158.9)}$ $= 0.687$	M1 A1 A1 *A1		
10	State both hypotheses (B0 for $r \dots$):	$H_0: \rho = 0, H_1: \rho \neq 0$	B1	4	
	State or use correct tabular two-tail r -value:	$r_{10, 5\%} = 0.632$	*B1		
	Valid method for reaching conclusion:	Reject H_0 if $ r >$ tabular value	M1		
	Correct conclusion (A.E.F, dep *A1, *B1):	There is non-zero correlation	A1		
10	Calculate gradient p in $x - \bar{x} = p(y - \bar{y})$:	$p = 18\,189.6 / 15\,158.9 = 1.20$ $x = 86.6 + 1.20(y - 63.9)$ $= 1.20y + 9.92$	B1	4	11
	Find regression line of x on y :		M1 A1	3	

Page 8	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

11 A	(i)	Use Pythagoras to find AB : Find $\angle SAB$:	$AB = \sqrt{(4a^2 + 12a^2)} = 4a$ $\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ or $\cos^{-1} 2a/4a$ or $\tan^{-1} 2a\sqrt{3}/2a$ $= 60^\circ$ so $\angle SAB = 30^\circ$	A.G.	M1 A1		
	(ii)	<i>EITHER</i> Resolve vertically and horizontally, e.g.: (F_A may be in either direction) Eliminate $N_B + F_A$ to find N_A : <i>OR</i>	$\frac{1}{2} N_A + \frac{1}{2} \sqrt{3} N_B + \frac{1}{2} \sqrt{3} F_A = W$ and $\frac{1}{2} \sqrt{3} N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$ $N_A = \frac{1}{2} W$	A.G.	M1 A1 A1	4	
	(iii)	Resolve in dirn. PQ to find N_A : Second resolution, e.g. in dirn. PS : Take moments, e.g. about A : (A1 for each side of eqn) Solve to find N_B : Use N_B to find F_A :	$N_A = \frac{1}{2} W$ $N_B + F_A = \frac{1}{2} \sqrt{3} W$ $\frac{1}{2} \sqrt{3} W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$ $= N_B \times 2a$ $N_B = \{(7\sqrt{3} - 6)/8\} W$ $F_A = \sqrt{3} N_A - N_B$ or $\frac{1}{2} \sqrt{3} W - N_B$ $= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.)	A.G.	(M1 A1) (A1) M1 A1 A1 M1 A1 M1 A1	3	
						7	14

Page 9	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – May/June 2014	9231	21

B	Estimate population variance:	$s_P^2 = (236 \cdot 0 - 42 \cdot 8^2 / 8) / 7$			
	(allow biased here: 0.8775 or 0.9367 ²)	$= 351 / 350$ or 1.003 or 1.001 ²	M1		
	Find confidence interval (allow z in place of t) e.g.:	$42 \cdot 8 / 8 \pm t \sqrt{s_P^2 / 8}$	M1		
	Use correct tabular t -value:	$t_{7, 0.975} = 2.365$	A1		
	Evaluate C.I. correct to 2 d.p.:	5.35 ± 0.84 or [4.51, 6.19]	A1	4	
	Formulate inequality for k (or equality for $k_{\max}$):	$(5.35 - k) / \sqrt{s_P^2 / 8} \geq [\text{or } >] t$	M1		
	Use correct tabular t -value:	$t_{7, 0.9} = 1.415$	A1		
	Solve for $k_{\max}$ (A0 if = or $\leq$ was used for k above):	$5.35 - k \geq 0.50, k_{\max} = 4.85$	A1	3	
	State hypotheses (B0 for $\bar{x} \dots$), e.g.:	$H_0: \mu_P = \mu_Q, H_1: \mu_P > \mu_Q$	B1		
	State assumption (A.E.F.):	Normal distns. for $[P \text{ and } Q]$ and equal variances	B1		
	Estimate (pooled) common variance:	$s^2 = (7 \times 1.003 + 11 \times 1.962) / 18$ $= 1.589$ or 1.261 ²	M1 A1		
	Calculate value of t (to 3 s.f.):	$t = (5.35 - 4.60) / (s \sqrt{(1/8 + 1/12)})$ $= 1.30$	M1 A1		
	Correct conclusion (A.E.F., $\sqrt{h}$ on t):	$t < t_{18, 0.9} = 1.33$ so Q's mean is not less than P's	B1 $\sqrt{h}$	7	14