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**FURTHER MATHEMATICS**

**9231/22**

Paper 2

**May/June 2019**

MARK SCHEME

Maximum Mark: 100

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **17** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.

**Abbreviations**

- AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- CWO Correct Working Only – often written by a ‘fortuitous’ answer
- ISW Ignore Subsequent Working
- SOI Seen or implied
- SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

| Question | Answer                                                          | Marks     | Guidance                                                             |
|----------|-----------------------------------------------------------------|-----------|----------------------------------------------------------------------|
| 1(i)     | $a_R = 2 (d\theta/dt)^2 = 2 (2 \sin 2t)^2 = 2 (2 \sin \pi/3)^2$ | <b>M1</b> | Verify radial acceleration $a_R$ at $t = \pi/6$ from $r\omega^2$     |
|          | $= 2 (\sqrt{3})^2 = 6 \text{ [m s}^{-2}\text{]}$ AG             | <b>A1</b> |                                                                      |
|          |                                                                 | <b>2</b>  |                                                                      |
| 1(ii)    | $a_T = 2 d^2\theta/dt^2 = 2 (4 \cos 2t) = 2 (4 \cos \pi/3)$     | <b>M1</b> | Find transverse acceleration $a_T$ at $t = \pi/6$ by differentiation |
|          | $= 4 \text{ [m s}^{-2}\text{]}$                                 | <b>A1</b> |                                                                      |
|          |                                                                 | <b>2</b>  |                                                                      |

| Question | Answer                                                                                                                                                        | Marks        | Guidance                                                                                                                                   |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| 2(i)     | $v_B = 2v_A, v_B^2 = 4 v_A^2, \omega^2 (a^2 - 1^2) = 4\omega^2 (a^2 - 3.5^2)$                                                                                 | <b>M1</b>    | Find amplitude $a$ : allow M1 for $v_A = 2 v_B$                                                                                            |
|          | $3 a^2 = 48, a = [\pm] 4 \text{ [m]}$                                                                                                                         | <b>A1</b>    |                                                                                                                                            |
|          | $1 = a\omega^2, \omega = 1/\sqrt{a} [= 1/2]$                                                                                                                  | <b>B1</b>    | Find $\omega$ from given maximum acceleration                                                                                              |
|          | $v_O = a\omega = \sqrt{a} = 2 \text{ [m s}^{-1}\text{]}$                                                                                                      | <b>B1</b>    | Find speed $v_O$ at $O$                                                                                                                    |
|          |                                                                                                                                                               | <b>4</b>     |                                                                                                                                            |
| 2(ii)    | $\omega t_{AB} = \sin^{-1} (3.5/a) + \sin^{-1}(1/a)$<br>$= \sin^{-1} 0.875 + \sin^{-1} 0.25$                                                                  | <b>M1 A1</b> | Find equation (AEF) for $\omega t_{AB}$ , combining $t_A$ and $t_B$ , using for example $x = a \sin \omega t$ allow sign errors for the M1 |
|          | $= 1.065 + 0.253 \text{ (or } t_{AB} = 2.131 + 0.505)$                                                                                                        | <b>A1</b>    | or $x = a \cos \omega t$                                                                                                                   |
|          | $t_{AB} = 2 \times 1.318 = 2.64 \text{ [s]}$                                                                                                                  | <b>A1</b>    | Hence find $t_{AB}$                                                                                                                        |
|          | <b>Alternative method for question 2(ii)</b>                                                                                                                  |              |                                                                                                                                            |
|          | $\omega t_{AB} = \cos^{-1} (-1/a) - \cos^{-1}(3.5/a)$<br>$= \cos^{-1} (-0.25) - \cos^{-1} 0.875$<br>or $\pi - \cos^{-1} 0.25 - \cos^{-1} 0.875 \text{ (AEF)}$ | <b>M1 A1</b> | or $x = a \cos \omega t$                                                                                                                   |
|          | $= 1.823 - 0.505 \text{ (or } t_{AB} = 3.647 - 1.011)$                                                                                                        | <b>A1</b>    |                                                                                                                                            |
|          | $t_{AB} = 2 \times 1.318 = 2.64 \text{ [s]}$                                                                                                                  | <b>A1</b>    | Hence find $t_{AB}$                                                                                                                        |
|          |                                                                                                                                                               | <b>4</b>     |                                                                                                                                            |

| Question | Answer                                                                                             | Marks     | Guidance                                                                          |
|----------|----------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------|
| 3(i)     | $2mv_A + 4mv_B = 4mu - 4mu$ <span style="float: right;"><math>[v_A + 2v_B = 0]</math> (AEF)</span> | <b>M1</b> | Use conservation of momentum for <i>A</i> and <i>B</i> ( <i>m</i> may be omitted) |
|          | $v_B - v_A = e(2u + u)$ <span style="float: right;"><math>[v_B - v_A = 3eu]</math></span>          | <b>M1</b> | Use Newton's restitution law with consistent LHS signs                            |
|          | $v_A = -2eu$ and $v_B = eu$                                                                        | <b>A1</b> | Combine to find $v_A$ and $v_B$ (A0 if directions unclear)                        |
|          |                                                                                                    | <b>3</b>  |                                                                                   |
| 3(ii)    | $[4mv_B'] + mv_C = 4mv_B - (4/3)mu$ (AEF)                                                          | <b>M1</b> | Use conservation of momentum for <i>B</i> & <i>C</i> ( <i>m</i> may be omitted)   |
|          | $v_C [-v_B'] = e(v_B + 4u/3)$                                                                      | <b>M1</b> | Use Newton's restitution law                                                      |
|          | $4v_B - (4/3)u = ev_B + 4eu/3, 4e - 4/3 = e^2 + 4e/3$                                              | <b>M1</b> | Combine to find quadratic equation for <i>e</i> using $v_B' = 0$                  |
|          | $3e^2 - 8e + 4 = 0, e = 2/3$ <span style="float: right;"><math>[v_C = 4u/3]</math></span>          | <b>A1</b> | Find value of <i>e</i> , (implicitly) rejecting $e = 2$                           |
|          |                                                                                                    | <b>4</b>  |                                                                                   |

| Question | Answer                                                                                                                                                                                       | Marks     | Guidance                                                              |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------|
| 3(iii)   | <p>For A: Loss = <math>\frac{1}{2} 2m(2u)^2 - \frac{1}{2} 2m(4/3u)^2 = (20/9) mu^2</math></p> <p>For B: Loss = <math>\frac{1}{2} 4m u^2 = \frac{1}{2} mu^2</math></p> <p>For C: Loss = 0</p> | <b>M1</b> | $v_A = -(4/3) u$ , $[v_B = (2/3) u]$ , $v_C = (4/3) u$<br>One correct |
|          |                                                                                                                                                                                              | <b>M1</b> | Other two correct                                                     |
|          | $E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$                                                                                                                  | <b>A1</b> | Hence find loss in KE                                                 |
|          | <b>Alternative method for question 3(iii)</b>                                                                                                                                                |           |                                                                       |
|          | $E_{\text{initial}} = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 + \frac{1}{2} m(4u/3)^2$<br>$= 4mu^2 + 2mu^2 + (8/9) mu^2 = (62/9) mu^2$                                                      | <b>M1</b> | Find initial KE of 3 particles in terms of $m$ and $u$                |
|          | $E_{\text{final}} = \frac{1}{2} 2mv_A^2 [+ \frac{1}{2} 4mv_B'^2] + \frac{1}{2} mv_C^2$<br>$= (16/9) mu^2 + (8/9) mu^2 = (24/9) mu^2$                                                         | <b>M1</b> | Find final KE of 3 particles in terms of $m$ and $u$                  |
|          | $E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$                                                                                                                  | <b>A1</b> | Hence find loss in KE                                                 |
|          | <b>Alternative method for question 3(iii)</b>                                                                                                                                                |           |                                                                       |
|          | $L_1 = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 - \frac{1}{2} 2mv_A'^2 - \frac{1}{2} 4mv_B'^2$<br>$= 4mu^2 + 2mu^2 - (16/9) mu^2 - (8/9) mu^2 = (30/9) mu^2$                                 | <b>M1</b> | Find losses in KE in both collisions in terms of $m$ and $u$          |
|          | $L_2 = \frac{1}{2} 4mv_B'^2 + \frac{1}{2} m(4u/3)^2 - [\frac{1}{2} 4mv_B'^2] - \frac{1}{2} mv_C'^2$<br>$= (8/9) mu^2 + (8/9) mu^2 - (8/9) mu^2 = (8/9) mu^2$                                 | <b>M1</b> |                                                                       |
|          | $E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$                                                                                                                  | <b>A1</b> | Hence find loss in KE                                                 |
|          |                                                                                                                                                                                              | <b>3</b>  |                                                                       |

| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks        | Guidance                                                                                                                                                |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4        | $A: T \times 5a/2 \sin(\pi - 2\theta) - W \times 2a \sin \theta = 0$<br>$C: F_A \times 5a/2 \sin \theta - R_A \times 5a/2 \cos \theta - W \times \frac{1}{2}a \sin \theta = 0$<br>$B: F_A \times 4a \sin \theta - R_A \times 4a \cos \theta - W \times 2a \sin \theta + T \times 3a/2 \sin(\pi - 2\theta) = 0$<br>$G: F_A \times 2a \sin \theta - R_A \times 2a \cos \theta - T \times \frac{1}{2}a \sin(\pi - 2\theta) = 0$<br>(G is mid-point of AB)<br>$D: R_A \times 5a \cos \theta - W \times 2a \sin \theta = 0$ | <b>M1 A1</b> | Take moments for rod about one chosen point<br>[ $\sin \theta = 2/\sqrt{5}$ , $\cos \theta = 1/\sqrt{5}$ , $\sin(\pi - 2\theta) = \sin 2\theta = 4/5$ ] |
|          | $R_A = T \sin \theta$<br>$F_A = W - T \cos \theta$                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <b>B1</b>    | Find two more independent equations                                                                                                                     |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <b>B1</b>    | e.g. resolution of forces on rod (a second moment equation may be used)                                                                                 |
|          | $F_A = \mu R_A$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <b>B1</b>    | Relate $F_A$ and $R_A$ (may be implied)                                                                                                                 |
|          | $T = (2W \sin \theta) / (5/2 \sin 2\theta) = 2W / (5 \cos \theta)$                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <b>M1</b>    | Find $T$ by any method (e.g. from moments about A)                                                                                                      |
|          | $= 2W/\sqrt{5}$ or $(2\sqrt{5}/5) W$ or $0.894 W$                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | <b>A1</b>    |                                                                                                                                                         |
|          | $F_A = (3/5) W$ and $R_A = (4/5) W$<br>$\mu = 3/4$ or $0.75$                                                                                                                                                                                                                                                                                                                                                                                                                                                           | <b>M1 A1</b> | Find or imply $F_A$ and $R_A$ by any method (e.g. from resolutions) and hence $\mu$                                                                     |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <b>A1</b>    |                                                                                                                                                         |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <b>10</b>    |                                                                                                                                                         |

| Question | Answer                                                                                                               | Marks        | Guidance                                                                                                                            |
|----------|----------------------------------------------------------------------------------------------------------------------|--------------|-------------------------------------------------------------------------------------------------------------------------------------|
| 5(i)     | $I_{rod} = \frac{1}{3} kMa^2$                                                                                        | <b>B1</b>    | Find or state MI of rod $AB$ about axis $L$                                                                                         |
|          | $I_{sphere} = \frac{2}{3} kM(2a)^2 + kM(3a)^2$ [= (35k/3) $Ma^2$ ]                                                   | <b>M1 A1</b> | M1 for one term correct, A1 for both terms correct                                                                                  |
|          | $I_{ring} = \frac{1}{2} \times Ma^2 + M(2a)^2$ [= (9/2) $Ma^2$ ]                                                     | <b>M1 A1</b> | M1 for one term correct, A1 for both terms correct                                                                                  |
|          | $I = (k/3 + 35k/3 + 9/2) Ma^2 = (3/2) (8k + 3) Ma^2$ AG                                                              | <b>A1</b>    | MI of object about axis $L$                                                                                                         |
|          |                                                                                                                      | <b>6</b>     |                                                                                                                                     |
| 5(ii)    | $[-] I d^2\theta/dt^2 = -kMg \times 3a \sin \theta + Mg \times 2a \sin \theta$<br>[ = - (3k - 2) $Mga \sin \theta$ ] | <b>M1 A1</b> | Use equation of circular motion to find $d^2\theta/dt^2$ where $\theta$ is angle of rod with vertical                               |
|          | $d^2\theta/dt^2 = - \{2g(3k - 2) / 3a(8k + 3)\} \theta$ (AEF)                                                        | <b>M1</b>    | Approximate $\sin \theta$ by $\theta$ to give standard form of SHM equation (M0 if wrong sign or $\cos \theta \approx \theta$ used) |
|          | SHM if $3k - 2 > 0$ , $k > 2/3$                                                                                      | <b>M1 A1</b> | Find possible values of $k$                                                                                                         |
|          | $T = 2\pi \sqrt{\{3a(8k + 3) / 2g(3k - 2)\}}$<br>or $\pi \sqrt{\{6a(8k + 3) / g(3k - 2)\}}$                          | <b>A1</b>    | Find period $T$                                                                                                                     |
|          |                                                                                                                      | <b>6</b>     |                                                                                                                                     |

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| Question | Answer                                                                 | Marks        | Guidance                                                         |
|----------|------------------------------------------------------------------------|--------------|------------------------------------------------------------------|
| 6(i)     | Mean = 3                                                               | <b>B1</b>    | State mean of $X$                                                |
|          |                                                                        | <b>1</b>     |                                                                  |
| 6(ii)    | $P(X=6) = q^5 p$ with $p = 1/3, q = 2/3$<br>$= 32/729$ or 0.0439 (AEF) | <b>B1</b>    | Find probability of score of 3 or 4 on exactly 6 throws          |
|          |                                                                        | <b>1</b>     |                                                                  |
| 6(iii)   | $P(X > 4) = q^4 = 16/81$ or 0.1975 or 0.198                            | <b>M1 A1</b> | Find probability of score of 3 or 4 on more than 4 throws        |
|          |                                                                        | <b>2</b>     |                                                                  |
| 6(iv)    | $1 - q^{n-1} < 0.95$ (AEF)                                             | <b>M1</b>    | Formulate condition for $n$ ( $1 - q^n$ is M0)                   |
|          | $0.05 < (2/3)^{n-1}, n - 1 < \log 0.05 / \log 2/3$                     | <b>M1</b>    | Set $q = 2/3$ , rearrange and take logs (any base) to give bound |
|          | $n - 1 < 7.39, n_{\max} = 8$                                           | <b>A1</b>    | Find $n_{\max}$ ( $>$ or $=$ can earn M1 M1 A0, max 2/3)         |
|          |                                                                        | <b>3</b>     |                                                                  |

| Question | Answer                                                                                    | Marks        | Guidance                                                         |
|----------|-------------------------------------------------------------------------------------------|--------------|------------------------------------------------------------------|
| 7(i)     | $F(x) = \int f(x) \, dx = -3/(4x) + x/4 [+c]$                                             | <b>M1</b>    | Find or state distribution function $F(x)$ for $1 \leq x \leq 3$ |
|          | $= -3/(4x) + x/4 + \frac{1}{2}$ or $\frac{1}{4}(-3/x + x + 2)$                            | <b>A1</b>    | using $F(1) = 0$ or $F(3) = 1$ to find $c$ if necessary          |
|          | $F(x) = 0$ ( $x < \text{or} \leq 1$ ), $F(x) = 1$ ( $x > \text{or} \geq 3$ )              | <b>A1</b>    | State $F(x)$ for other values of $x$                             |
|          |                                                                                           | <b>3</b>     |                                                                  |
| 7(ii)    | $\int_1^Q f(x) \, dx = -3/4Q + Q/4 + \frac{1}{2} = \frac{1}{4}$ [or $\frac{3}{4}$ ] (AEF) | <b>M1</b>    | Formulate equation for either quartile value $Q$                 |
|          | $Q^2 + [\text{or} -] Q - 3 = 0$                                                           | <b>A1</b>    |                                                                  |
|          | $Q_1 = \frac{1}{2}(-1 + \sqrt{13})$ , $Q_3 = \frac{1}{2}(1 + \sqrt{13})$                  | <b>A1 A1</b> | Find lower quartile $Q_1$ and upper quartile $Q_3$               |
|          | $Q_3 - Q_1 = 1$                                                                           | <b>A1</b>    | Find interquartile range                                         |
|          |                                                                                           | <b>5</b>     |                                                                  |

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| Question | Answer                                                                                           | Marks        | Guidance                                                                                                    |
|----------|--------------------------------------------------------------------------------------------------|--------------|-------------------------------------------------------------------------------------------------------------|
| 8        | $H_0: \mu_b - \mu_e = 0.05, H_1: \mu_b - \mu_e > 0.05$ (AEF)                                     | <b>B1</b>    | State both hypotheses (B0 for $\bar{x} \dots$ )                                                             |
|          | $d_i: 0.10 \ 0.07 \ 0.03 \ 0.03 \ 0.21 \ 0.19$ (or in sec)                                       | <b>M1</b>    | Consider differences $d_i$ from e.g. $x_b - y_e$                                                            |
|          | $\bar{d} = 0.63 / 6 = 0.105$ (or 6.3 sec)                                                        | <b>B1</b>    | Find sample mean                                                                                            |
|          | $s^2 = (0.0969 - 0.63^2/6) / 5$<br>[ = 123/20 000 or 0.00615 or 0.0784 <sup>2</sup> ] (or 22.14) | <b>M1</b>    | Estimate population variance<br>(allow biased here: [41/8000 or 0.005125 or 0.0716 <sup>2</sup> ])          |
|          | $t_{5, 0.9} = 1.476$ (to 3 sf)                                                                   | <b>B1</b>    | State or use correct tabular $t$ -value                                                                     |
|          | $t = (\bar{d} - 0.05) / (s/\sqrt{6}) = 1.72$                                                     | <b>M1 A1</b> | Find value of $t$<br>(or compare $\bar{d} - 0.05 = 0.055$ with $(t_{5, 0.9})s/\sqrt{6} = 0.0473$ )          |
|          | [Reject $H_0$ :] Evidence for organiser's belief<br>or times improve by more than 0.05 min (AEF) | <b>B1</b>    | <b>FT</b> on both $t$ -values<br>Consistent conclusion                                                      |
|          |                                                                                                  |              | <b>SC</b> Wrong type of hypothesis test can earn only<br>B1 for hypotheses<br>B1FT for conclusion (max 2/8) |
|          |                                                                                                  | <b>8</b>     |                                                                                                             |

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| Question | Answer                                                                                                 | Marks        | Guidance                                                                                                             |
|----------|--------------------------------------------------------------------------------------------------------|--------------|----------------------------------------------------------------------------------------------------------------------|
| 9 (i)    | $E_3 = (3/16) \int_{1.6}^{2.4} (4-x)^{1/2} dx$<br>$= (3/16) [ - (2/3) (4-x)^{3/2} ]_{1.6}^{2.4}$       | <b>M1</b>    | State or imply expression for required expected value $E_3$ of $X$                                                   |
|          | $= (2.4^{3/2} - 1.6^{3/2}) / 8 = 1.694/8$ or 0.2118                                                    | <b>A1</b>    | Find expected value $E_3$ (may be implied in finding 50 $E_3$ )<br>(M1 A1 requires adequate explicit working)        |
|          | 50 $E_3 = 10.59$ AG                                                                                    | <b>A1</b>    | Hence verify corresponding expected frequency                                                                        |
|          |                                                                                                        | <b>3</b>     |                                                                                                                      |
| 9(ii)    | $H_0$ : Distribution fits data (AEF)                                                                   | <b>B1</b>    | State (at least) null hypothesis in full                                                                             |
|          | $O_i$ 18 16 8 <u>8</u>                                                                                 | <b>M1</b>    | Combine values consistent with all exp. values $\geq 5$                                                              |
|          | $E_i$ : 14.22 12.54 10.59 <u>12.65</u>                                                                 | <b>M1 A1</b> | Find value of $X^2$ from $\Sigma (E_i - O_i)^2 / E_i$ [or $\Sigma O_i^2 / E_i - n$ ]                                 |
|          | $X^2 = 1.005 + 0.955 + 0.633 + 1.709 = 4.30$                                                           |              |                                                                                                                      |
|          | No. $n$ of cells: 5 <u>4</u> 3<br>$\chi_{n-1, 0.95}^2$ : 9.488 <u>7.815</u> 5.991 (to 3 s.f.)          | <b>B1</b>    | <b>FT</b> on number, $n$ , of cells used to find $X^2$<br>State or use consistent tabular value $\chi_{n-1, 0.95}^2$ |
|          | Accept $H_0$ if $X^2 < \text{tabular value}$ (AEF)                                                     | <b>M1</b>    | State or imply valid method for conclusion                                                                           |
|          | 4.30 [ $\pm 0.01$ ] < 7.81[5] so distribution fits [data]<br>or distribution is a suitable model (AEF) | <b>A1</b>    | Conclusion (requires both values approx. correct)                                                                    |
|          |                                                                                                        | <b>7</b>     |                                                                                                                      |

| Question | Answer                                                                                                                                                 | Marks        | Guidance                                                                                                           |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|--------------------------------------------------------------------------------------------------------------------|
| 10(i)    | $\Sigma x = 25, \Sigma y = 25 + q, \Sigma xy = 135 + 4q,$<br>$\Sigma x^2 = 141, \Sigma y^2 = 159 + q^2$                                                | <b>B1</b>    | Find required values                                                                                               |
|          | $S_{xy} = 135 + 4q - 25(25 + q)/5 = [10 - q]$ (AEF)<br>[ $S_{xx} = 141 - 25^2/5 = 16$ ]<br>$S_{yy} = (159 + q^2) - (25 + q)^2/5 [= 34 - 10q + 4q^2/5]$ | <b>M1 A1</b> | ( $S_{xy}, S_{xx}, S_{yy}$ may be scaled by the same constant)                                                     |
|          | $40 - 4q = 170 - 50q + 4q^2, 2q^2 - 23q + 65 = 0$<br>$(2q - 13)(q - 5) = 0, q = 5$                                                                     | <b>M1 A1</b> | Equate gradient 5/4 in line of $x$ on $y$ to $S_{xy} / S_{yy}$<br>and solve quadratic to find integer value of $q$ |
|          |                                                                                                                                                        | <b>5</b>     |                                                                                                                    |
| 10(ii)   | $c = 25/5 - (5/4)(25 + q)/5 = -(5 + q) / 4$                                                                                                            | <b>M1 A1</b> | Find $c$ from $\bar{x} - (5/4) \bar{y}$                                                                            |
|          | $= -5/2$ or $-2.5$                                                                                                                                     | <b>A1</b>    |                                                                                                                    |
|          |                                                                                                                                                        | <b>3</b>     |                                                                                                                    |
| 10(iii)  | $r = S_{xy} / \sqrt{(S_{xx} S_{yy})} = 5 / \sqrt{(16 \times 4)}$                                                                                       | <b>M1 A1</b> | Find correlation coefficient $r$                                                                                   |
|          | $= 5/8$ or $0.625$                                                                                                                                     | <b>A1</b>    |                                                                                                                    |
|          |                                                                                                                                                        | <b>3</b>     |                                                                                                                    |

| Question | Answer                                                                                                                                                                                | Marks        | Guidance                                                                                                                                                  |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11E(i)   | $\frac{1}{2}mu_P^2 = \frac{1}{2}m(21ag/2) + mga \quad [u_P^2 = (25/2)ag]$                                                                                                             | <b>M1 A1</b> | $u_P$ is speed of $P$ at lowest point, $v_Q$ is speed of $Q$ immediately after collision.<br>Apply conservation of energy at lowest point (A0 if no $m$ ) |
|          | $\frac{1}{2}4mv_Q^2 = 4mag$                                                                                                                                                           | <b>M1</b>    | Find speed $v_Q$ at lowest point by conservation of energy (A0 if no $m$ )                                                                                |
|          | $v_Q = \sqrt{2ag}$ or $1.41\sqrt{ag}$ or $4.47\sqrt{a}$                                                                                                                               | <b>A1</b>    |                                                                                                                                                           |
|          | $mu_P = [\pm]mv_P + 4mv_Q$                                                                                                                                                            | <b>M1</b>    | Find $v_P$ using conservation of momentum ( $m$ may be omitted)                                                                                           |
|          | $v_P = [\pm](-5/\sqrt{2} + 4\sqrt{2})\sqrt{ag}$                                                                                                                                       | <b>A1</b>    |                                                                                                                                                           |
|          | $ v_P  = (3/\sqrt{2})\sqrt{ag}$ or $2.12\sqrt{ag}$ or $0.671\sqrt{a}$ (AEF)                                                                                                           | <b>A1</b>    | Hence find speed of $P$                                                                                                                                   |
|          |                                                                                                                                                                                       | <b>7</b>     |                                                                                                                                                           |
| 11E(ii)  | $V_P$ is speed of $P$ when it loses contact<br>$\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga(1 + \cos \alpha)$<br>$[V_P^2 = (9/2)ag - 2ga(1 + \cos \alpha) = (5/2 - 2 \cos \alpha)ag]$ | <b>M1 A1</b> | Apply conservation of energy at $D$ (A0 if no $m$ )                                                                                                       |
|          | $[R_D =] mV_P^2/a - mg \cos \alpha = 0 \quad [V_P^2 = ag \cos \alpha]$                                                                                                                | <b>M1 A1</b> | Apply $F = ma$ radially at $D$ with reaction = 0                                                                                                          |
|          | $(5/2 - 2 \cos \alpha)ag = ag \cos \alpha, \cos \alpha = 5/6$ or $0.833$                                                                                                              | <b>A1</b>    | Combine to find $\cos \alpha$                                                                                                                             |
|          |                                                                                                                                                                                       | <b>5</b>     |                                                                                                                                                           |

| Question | Answer                                                                                              | Marks        | Guidance                                                                                                                                                                                                                            |
|----------|-----------------------------------------------------------------------------------------------------|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11O(i)   | $ts_A / \sqrt{8} = \frac{1}{2} (16.7 - 13.5) [= 1.6]$                                               | <b>M1</b>    | Relate $s_A$ to semi-width of confidence interval                                                                                                                                                                                   |
|          | $t_{7, 0.975} = 2.365$ (to 3 s.f.)                                                                  | <b>A1</b>    | State or use correct tabular $t$ value                                                                                                                                                                                              |
|          | $[s_A = \sqrt{8} \times 1.6 / 2.365 = 1.9135], s_A^2 = 3.66[16]$                                    | <b>A1</b>    | Hence find unbiased estimate of $A$ 's population variance                                                                                                                                                                          |
|          |                                                                                                     | <b>3</b>     |                                                                                                                                                                                                                                     |
| 11O(ii)  | $H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B$ (AEF)                                                      | <b>B1</b>    | State hypotheses (B0 for $\bar{x} \dots$ )                                                                                                                                                                                          |
|          | $[\bar{x}_A = 15.1], \bar{x}_B = 85.2 / 6 = 14.2$                                                   | <b>B1</b>    | Find sample mean for $B$                                                                                                                                                                                                            |
|          | $s_B^2 = (1221.06 - 85.2^2/6) / 5$<br>$= 561/250$ or $2.244$ or $1.498^2$ (all to 3 s.f.)           | <b>M1</b>    | Estimate or imply population variance for $B$<br>(allow biased here: $1.87$ or $1.367^2$ )                                                                                                                                          |
|          | $s^2 = (7 s_A^2 + 5 s_B^2) / 12 = 3.0709$ or $1.752^2$                                              | <b>M1 A1</b> | Estimate (pooled) common variance ( $s_B^2$ not needed explicitly)                                                                                                                                                                  |
|          | $t_{12, 0.95} = 1.782$                                                                              | <b>B1</b>    | State or use correct tabular $t$ value                                                                                                                                                                                              |
|          | $[-] t = (\bar{x}_A - \bar{x}_B) / (s \sqrt{(1/8 + 1/6)}) = 0.951$<br>$t < 1.78$ so [accept $H_0$ ] | <b>M1 A1</b> | Find value of $t$ (or can compare $\bar{x}_A - \bar{x}_B = 0.9$ with 1.69)<br>Correct conclusion                                                                                                                                    |
|          | mean mass of $B$ not less than mean mass of $A$ (AEF)                                               | <b>B1</b>    |                                                                                                                                                                                                                                     |
|          |                                                                                                     | <b>9</b>     |                                                                                                                                                                                                                                     |
|          |                                                                                                     |              | <b>SC1:</b> Implicitly taking $s_A^2, s_B^2$ as unequal population variances<br>(may also earn first B1 B1 M1)<br>$z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A^2/8 + s_B^2/6)}$<br>$= 0.9 / \sqrt{(0.8317)} = 0.987$<br>$z < 1.645$ so |
|          |                                                                                                     |              | <b>DepSC1:</b> mean mass of $B$ not less than mean mass of $A$ (AEF)<br>Comparison with $z_{0.95}$ and conclusion (FT on $z$ )<br>(can earn at most 5/9)                                                                            |