

Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 The cubic equation $7x^3 + 3x^2 + 5x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$. [3]

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(b) Find the value of $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$. [2]

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(c) Find the value of $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$. [2]

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2 The sequence $u_1, u_2, u_3, \dots$ is such that $u_1 = 1$ and $u_{n+1} = 2u_n + 1$ for $n \geq 1$.

(a) Prove by induction that $u_n = 2^n - 1$ for all positive integers n . [5]

[illegible]

(b) Deduce that u_{2^n} is divisible by u_n for $n \geq 1$. [2]

[illegible]

3 Let $S_n = 2^2 + 6^2 + 10^2 + \dots + (4n-2)^2$.

(a) Use standard results from the List of Formulae (MF19) to show that $S_n = \frac{4}{3}n(4n^2 - 1)$. [4]

[illegible]

- (b) Express $\frac{n}{S_n}$ in partial fractions and find $\sum_{n=1}^N \frac{n}{S_n}$ in terms of N . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) Deduce the value of $\sum_{n=1}^{\infty} \frac{n}{S_n}$. [1]

4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular.

[3]

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The matrices $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}.$

(b) Find the value of k .

[3]

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- (c) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **CAB**. [5]

[illegible]

- 5 The curve C has polar equation $r = a \tan \theta$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{4}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

- (b)** Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{4}\pi$. [4]

[illegible]

- (c) Show that C has Cartesian equation $y = \frac{x^2}{\sqrt{a^2 - x^2}}$. [3]

[illegible]

- (d) Using your answer to part (b), deduce the exact value of $\int_0^{\frac{1}{2}a\sqrt{2}} \frac{x^2}{\sqrt{a^2 - x^2}} \, dx$. [2]

This image shows a full page of a handwriting practice worksheet. It consists of ten sets of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

6 The curve C has equation $y = \frac{10+x-2x^2}{2x-3}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Show that C has no turning points. [3]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

- (d) Sketch the curve with equation $y = \left| \frac{10+x-2x^2}{2x-3} \right|$ and find in exact form the set of values of x for which $\left| \frac{10+x-2x^2}{2x-3} \right| < 4$. [6]

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- 7 The lines l_1 and l_2 have equations $\mathbf{r} = -5\mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{j} + \mathbf{k})$ respectively. The plane Π contains l_1 and is parallel to l_2 .

(a) Find the equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the distance between l_2 and Π . [3]

[illegible]

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

- (c) Show that P has position vector $\frac{55}{27}\mathbf{i} - 5\mathbf{j} + \frac{22}{27}\mathbf{k}$ and state a vector equation for PQ . [8]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

[illegible]

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