



CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

9231/21

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

BLANK PAGE

1 Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$.

[6]

[illegible]

2 It is given that $y = 2^x$.

- (a) By differentiating $\ln y$ with respect to x , show that $\frac{dy}{dx} = 2^x \ln 2$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Write down $\frac{d^2y}{dx^2}$. [1]

.....

.....

.....

- (c) Hence find the first three terms in the Maclaurin's series for 2^x . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

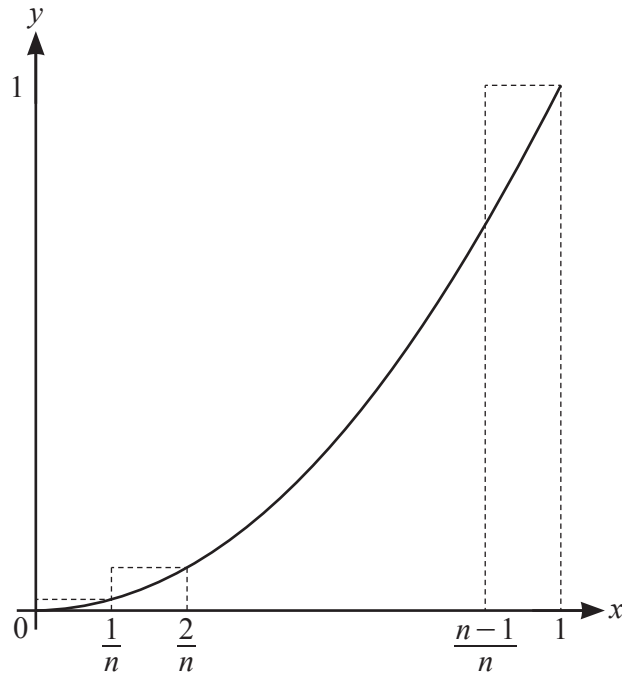
- 3 (a)** Find the roots of the equation $z^3 = -1 - i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Let $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$, where k is a positive integer and z_1, z_2, z_3 are the roots of $z^3 = -1 - i$.

- (b) Express w in the form $Re^{i\alpha}$, where $R > 0$, giving R and α in terms of k . [3]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

[illegible]

- (b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 \, dx$. [4]

5 The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intersect at the point where $x = a$.

(a) Find the exact value of a , giving your answer in logarithmic form. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Sketch C_1 and C_2 on the same diagram. [2]

[Turn over

6 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}n} dx$.

(a) Find the exact value of I_1 .

[2]

.....

.....

.....

.....

.....

(b) By considering $\frac{d}{dx} \left(x(1-x^2)^{-\frac{1}{2}n} \right)$, or otherwise, show that

$$nI_{n+2} = 2^{n-1}3^{-\frac{1}{2}n} + (n-1)I_n.$$

[5]

[illegible]

- (c) Find the exact value of I_5 giving the answer in the form $k\sqrt{3}$, where k is a rational number to be determined. [3]

[illegible]

7 It is given that $x = t^3y$ and

$$t^3 \frac{d^2 y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 61e^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}. \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[Turn over

- 8 (a)** Find the values of a for which the system of equations

$$3x + y + z = 0,$$

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution.

[3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (b)** Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$.

[4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

© UCLES 2020

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.