



CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

9231/23

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}. \quad [6]$$

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x} dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact value of I_2 . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

3 The matrix $\mathbf{A}$ is given by

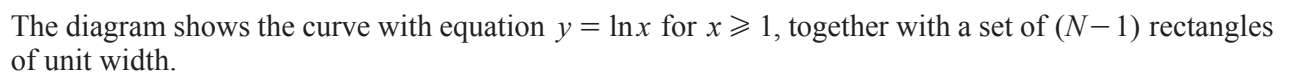
$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.

[4]

[illegible]

[Turn over


$$\ln N! > N \ln N - N + 1. \quad [5]$$
[illegible]

- (b)** Use a similar method to find, in terms of N , an upper bound for $\ln N!$. [3]

[illegible]

5 The curve C has parametric equations

$$x = \frac{1}{2}t^2 - \ln t, \quad y = 2t + 1, \quad \text{for } \frac{1}{2} \leq t \leq 2.$$

(a) Find the exact length of C .

[5]

[illegible]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a)** Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta. \quad [3]$$

[illegible]

The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

- (b)** By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x , show that

$$\frac{dy}{dx} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right). \quad [4]$$

[illegible]

- (c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\cos\left(x+\frac{1}{4}\pi\right)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

is $x + \sqrt{x^2 + 1}$.

[4]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 20 evenly spaced, horizontal dotted lines running across the width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

(b) Hence find the solution of the differential equation

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

[illegible]

- 8 (a)** Use de Moivre's theorem to show that $\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10)$. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

It is given that $\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$.

- (b)** Find the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^6\left(\frac{1}{4}x\right) + \sin^6\left(\frac{1}{4}x\right) \right) dx$. [4]

This image shows a single sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

-
- This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.