

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

- 1 A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O . The tension in the string is $4mg$.

Find, in terms of a and g , the time that P takes to make one complete revolution.

[2]

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- 2 A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where v ms⁻¹ is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g , k and t .

[6]

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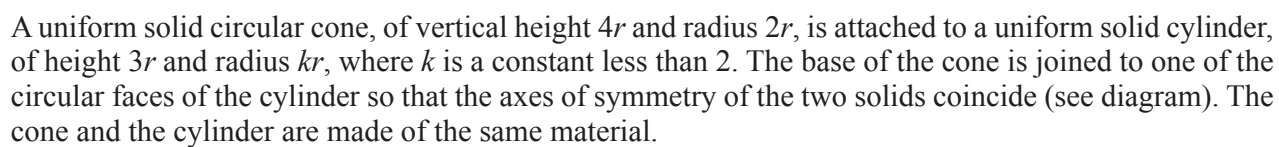
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- 3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a . The particle moves in complete vertical circles about O . The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O . The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A:T_B$. [6]

[illegible]

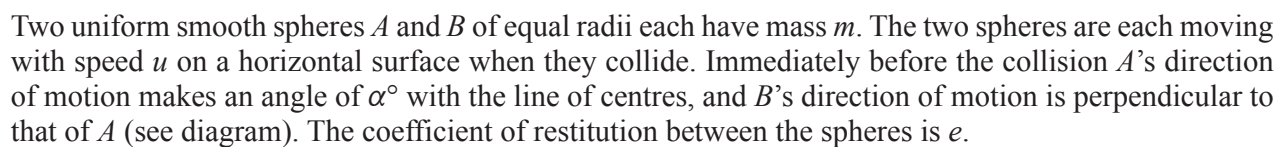


- [illegible]

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan \alpha = \frac{1}{8}$.

- (b)** Given that the centre of mass of the combined solid is within the cylinder, find the value of k . [4]

[illegible]



(a) Show that $\tan \alpha = \frac{1+e}{1-e}$. [4]

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[Turn over

- 6 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

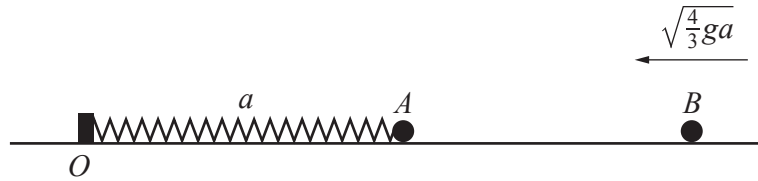
(a) Show that $\tan \alpha = \frac{1}{2} \tan \theta$.

[6]

[illegible]

- (b)** Given that $\tan \theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O . The other end of the spring is attached to a particle A of mass km , where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a . A second particle B , of mass m , is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

- (a)** Find the value of k .

[6]

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[4]

[illegible]

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