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9231/22

May/June 2021

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

[Turn over

- 1 (a)** Given that a is an integer, show that the system of equations

$$ax + 3y + z = 14,$$

$$2x + y + 3z = 0,$$

$$-x + 2y - 5z = 17,$$

has a unique solution and interpret this situation geometrically.

[4]

[illegible]

- (b)** Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equations in part **(a)**. [1]

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2 The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x + 1.$$

(a) Find the general solution for y in terms of x .

[6]

[illegible]

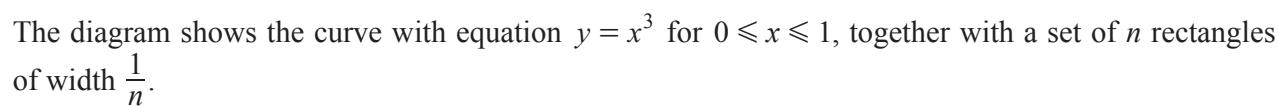
(b) State an approximate solution for large positive values of x .

$$[1]$$

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- $$U_n = \left(\frac{n+1}{2n}\right)^2. \quad [4]$$

[illegible]

- (b)** Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 x^3 dx$. [4]

[illegible]

- (c) Find the least value of n such that $U_n - L_n < 10^{-3}$. [2]

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- 4** Find the solution of the differential equation

$$\sin \theta \frac{dy}{d\theta} + y = \tan \frac{1}{2} \theta,$$

where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [9]

[You may use without proof the result that $\int \operatorname{cosec} \theta \, d\theta = \ln \tan \frac{1}{2} \theta$.]

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- 5 (a)** State the sum of the series $z+z^2+z^3+\dots+z^n$, for $z \neq 1$. [1]

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- (b)** Given that z is an n th root of unity and $z \neq 1$, deduce that $1 + z + z^2 + \dots + z^{n-1} = 0$. [2]

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- (c) Given instead that $z = \frac{1}{3}(\cos \theta + i \sin \theta)$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3 \cos \theta - 1}{10 - 6 \cos \theta} . \quad [7]$$

[illegible]

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^3$. [4]

- 7 (a) It is given that $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$.

Express $\cosh y$ in terms of x and hence show that $\sinh y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$. [3]

[illegible]

- (b)** Find the first three terms in the Maclaurin's series for $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ in the form

$$\ln a + bx + cx^2,$$

where a , b and c are constants to be determined. [7]

[illegible]

[6]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running across the width of the page. There are no margins, text, or other markings present.

[illegible]

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