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9231/11

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Let a be a positive constant.

- (a) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(ar+1)(ar+a+1)}$ in terms of n and a . [4]

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- (b) Find the value of a for which $\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}$. [3]

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2 The points A, B, C have position vectors

$$4\mathbf{i} - 4\mathbf{j} + \mathbf{k}, \quad -4\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}, \quad 4\mathbf{i} - \mathbf{j} - 2\mathbf{k},$$

respectively, relative to the origin O .

- (a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Find the perpendicular distance from O to the plane ABC . [2]

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- (c) The point D has position vector $2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$.

Find the coordinates of the point of intersection of the line OD with the plane ABC . [3]

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- 3** The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 > 4$ and, for $n \geq 1$,

$$u_{n+1} = \frac{u_n^2 + u_n + 12}{2u_n}.$$

- (a) By considering $u_{n+1} - 4$, or otherwise, prove by mathematical induction that $u_n > 4$ for all positive integers n . [5]

[illegible]

(b) Show that $u_{n+1} < u_n$ for $n \geq 1$. [3]

4 The cubic equation $2x^3 + 5x^2 - 6 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$. [3]

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(b) Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$. [3]

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- (c) Find also the value of $\frac{1}{\alpha^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9}$. [2]

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5 The curve C has equation $y = \frac{2x^2 - x - 1}{x^2 + x + 1}$.

- (a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote of C . [3]

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- (b) Find the coordinates of the stationary points on C . [4]

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- (c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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- (d) Sketch the curve with equation $y = \left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right|$ and state the set of values of k for which $\left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right| = k$ has 4 distinct real solutions.

[2]

- 6** The curve C has polar equation $r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right)$, where $0 \leq \theta \leq 2$.

- (a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

- (b) Find the exact value of the area of the region bounded by C and the half-line $\theta = 2$. [5]

(c) Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$,

and verify that this equation has a root between 0.6 and 0.7. [5]

[illegible]

7 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

- (a) Find the set of values of k for which $\mathbf{A}$ is non-singular. [3]

[illegible]

- (b)** Given that $\mathbf{A}$ is non-singular, find, in terms of k , the entries in the top row of $\mathbf{A}^{-1}$. [4]

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- (c) Given that $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, give an example of a matrix $\mathbf{C}$ such that $\mathbf{BAC} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$. [4]

[illegible]

- (d) Find the set of values of k for which the transformation in the x - y plane represented by $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ has two distinct invariant lines through the origin. [6]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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