



CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

9231/13

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

- 1 (a) Sketch the curve with equation $y = \frac{x+1}{x-1}$. [2]

- (b) Sketch the curve with equation $y = \frac{|x|+1}{|x|-1}$ and find the set of values of x for which $\frac{|x|+1}{|x|-1} < -2$. [4]

.....

.....

.....

.....

2 The cubic equation $x^3 + 5x^2 + 10x - 2 = 0$ has roots α, β, γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [3]

[illegible]

(b) Show that the matrix $\begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$ is singular. [4]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.

3 A curve C has equation $y = \frac{ax^2 + x - 1}{x - 1}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that there is no point on C for which $1 < y < 1 + 4a$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Sketch *C*. You do not need to find the coordinates of the intersections with the axes. [3]

4 Let $u_r = e^{rx}(e^{2x} - 2e^x + 1)$.

(a) Using the method of differences, or otherwise, find $\sum_{r=1}^n u_r$ in terms of n and x . [3]

[illegible]

(b) Deduce the set of non-zero values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

[illegible]

- (c) Using a standard result from the list of formulae (MF19), find $\sum_{r=1}^n \ln u_r$ in terms of n and x . [3]

5 Let $\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where a is a positive constant.

(a) State the type of the geometrical transformation in the x - y plane represented by $\mathbf{A}$. [1]

(b) Prove by mathematical induction that, for all positive integers n ,

$$\mathbf{A}^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}. \quad [5]$$

Let $\mathbf{B} = \begin{pmatrix} b & b \\ a^{-1} & a^{-1} \end{pmatrix}$, where b is a positive constant.

- (c) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{A}^n \mathbf{B}$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 The curve C has Cartesian equation $x^2 + xy + y^2 = a$, where a is a positive constant.

(a) Show that the polar equation of C is $r^2 = \frac{2a}{2 + \sin 2\theta}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Sketch the part of C for $0 \leq \theta \leq \frac{1}{4}\pi$. [2]

The region R is enclosed by this part of C , the initial line and the half-line $\theta = \frac{1}{4}\pi$.

(c) It is given that $\sin 2\theta$ may be expressed as $\frac{2 \tan \theta}{1 + \tan^2 \theta}$. Use this result to show that the area of R is

$$\frac{1}{2}a \int_0^{\frac{1}{4}\pi} \frac{1 + \tan^2 \theta}{1 + \tan \theta + \tan^2 \theta} d\theta$$

and use the substitution $t = \tan \theta$ to find the exact value of this area.

[8]

[illegible]

7 The position vectors of the points A, B, C, D are

$$7\mathbf{i}+4\mathbf{j}-\mathbf{k}, \quad 11\mathbf{i}+3\mathbf{j}, \quad 2\mathbf{i}+6\mathbf{j}+3\mathbf{k}, \quad 2\mathbf{i}+7\mathbf{j}+\lambda\mathbf{k}$$

respectively.

- (a) Given that the shortest distance between the line AB and the line CD is 3, show that $\lambda^2 - 5\lambda + 4 = 0$. [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Let Π_1 be the plane ABD when $\lambda = 1$.

Let Π_2 be the plane ABD when $\lambda = 4$.

- (b) (i)** Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [2]

.....

.....

.....

.....

.....

- (ii) Find an equation of Π_2 , giving your answer in the form $ax + by + cz = d$. [4]

[illegible]

9231/13/M/J/22

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no margins, text, or other markings.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.