



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/21

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 The curve C has polar equation $r = e^{\frac{3}{4}\theta}$ for $0 \leq \theta \leq \alpha$.

Given that the length of C is s , find α in terms of s .

[5]

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- 2 (a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2 \sinh^2 x + 1. \quad [3]$$

- (b) Find the set of values of k for which $\cosh 2x = k \sinh x$ has two distinct real roots. [5]

3 The variables t and x are related by the differential equation

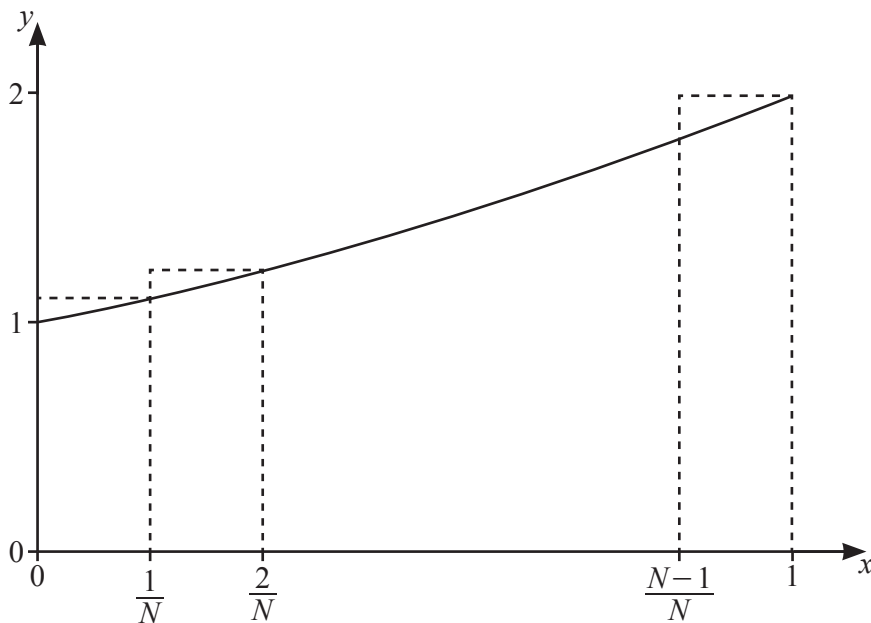
$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for x in terms of t . [6]

[illegible]

- (b) Deduce an approximate value of $\frac{d^2x}{dt^2}$ for large positive values of t . [2]

- 4 The diagram shows the curve with equation $y = 2^x$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.



- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 2^x dx < U_N$, where

$$U_N = \frac{2^{\frac{1}{N}}}{N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

[illegible]

- (b)** Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 2^x dx$. [4]

[illegible]

- (c) Find the least value of N such that $U_N - L_N < 10^{-4}$. [2]

[illegible]

5 The variables x and y are such that $y = 0$ when $x = 0$ and

$$(x+1)y + (x+y+1)^3 = 1.$$

(a) Show that $\frac{dy}{dx} = -\frac{3}{4}$ when $x = 0$. [3]

[illegible]

(b) Find the Maclaurin's series for y up to and including the term in x^2 . [7]

[illegible]

- 6** Use the substitution $y = vx$ to find the solution of the differential equation

$$x \frac{dy}{dx} = y + \sqrt{9x^2 + y^2}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial in x .
[10]

[illegible]

7 (a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 7\theta = \frac{\operatorname{cosec}^7 \theta}{7 \operatorname{cosec}^6 \theta - 56 \operatorname{cosec}^4 \theta + 112 \operatorname{cosec}^2 \theta - 64}.$$

[6]

[illegible]

(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\operatorname{cosec} q\pi$, where q is rational.

[5]

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- 8 (a)** Find the value of a for which the system of equations

$$3x + ay = 0,$$

$$5x - y = 0,$$

$$x + 3y + 2z = 0,$$

does not have a unique solution.

[2]

[illegible]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

- (b)** Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$.

[7]

[illegible]

- (c) Use the characteristic equation of $\mathbf{A}$ to show that

$$(\mathbf{A} + 6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

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