Cambridge Assessment International Education

# Cambridge International AS & A Level

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| CANDIDATE NAME | | | |
| CENTRE NUMBER | | CANDIDATE NUMBER | |

*2464209347*

**FURTHER MATHEMATICS** **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2022**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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**1** Find the roots of the equation $z^3 = 7\sqrt{3} - 7\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

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**2** **(a)** Find the coefficient of $x^2$ in the Maclaurin's series for $-\ln\cos x$. [4]

**(b)** Find the length of the arc of the curve with equation $y = -\ln\cos x$ from the point where $x = 0$ to the point where $x = \frac{1}{4}\pi$. [4]

**3** The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

**(a)** Find the eigenvalues of **A**. [4]

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**(b)** Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where $p$ and $q$ are constants to be determined. [3]

**4** It is given that

$$x = -t + \tan^{-1} t \qquad \text{and} \qquad y = t + \sinh^{-1} t.$$

**(a)** Show that $\dfrac{dy}{dx} = -\dfrac{t^2 + 1 + \sqrt{t^2 + 1}}{t^2}$. [4]

**(b)** Find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{3}{4}$. [5]

**5** Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

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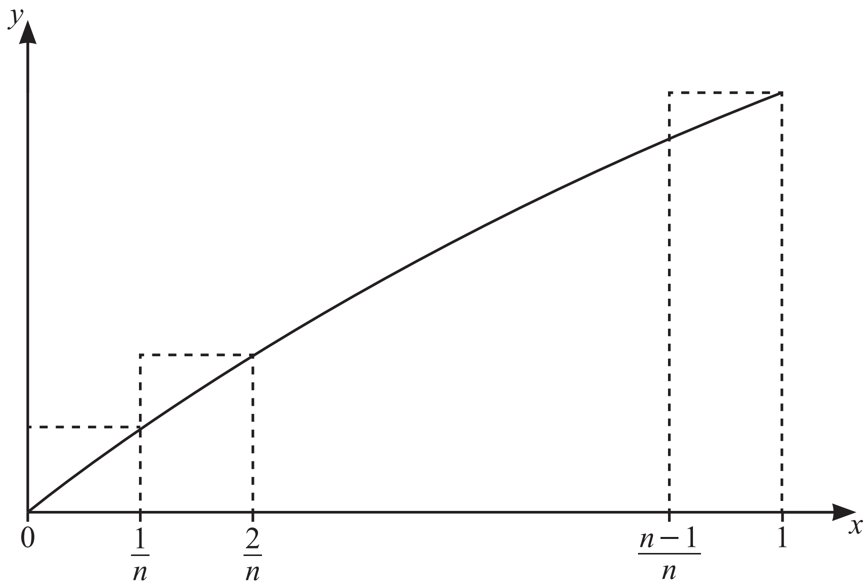


The diagram shows the curve with equation $y = \ln(1+x)$ for $0 \leqslant x \leqslant 1$, together with a set of $n$ rectangles each of width $\frac{1}{n}$.

**(a)** By considering the sum of the areas of these rectangles, show that $\int_0^1 \ln(1+x)\,dx < U_n$, where

$$U_n = \frac{1}{n}\ln\frac{(2n)!}{n!} - \ln n.$$

[4]

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**(b)** Use a similar method to find, in terms of $n$, a lower bound $L_n$ for $\int_0^1 \ln(1+x)\,dx$. [4]

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**(c)** By simplifying $U_n - L_n$, show that $\lim_{n\to\infty}(U_n - L_n) = 0$. [2]

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7 The variables $x$ and $y$ are related by the differential equation

$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

**(a)** Find $y$ in terms of $x$. [8]

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**(b)** Deduce the exact value of $x$ for which $y = 0$. Give your answer in logarithmic form. [3]

**8** **(a)** Find $\int \sin\theta \cos^n\theta \, d\theta$, where $n \neq -1$. [2]

Let $I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m\theta \cos^n\theta \, d\theta$.

**(b)** Show that, for $m \geqslant 2$ and $n \geqslant 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}.$$ [5]

**(c)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)^5$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta,$$

where $a$, $b$ and $c$ are constants to be determined. [5]

**(d)** Using the results given in parts **(b)** and **(c)**, find the exact value of $I_{2,5}$. [4]

**Additional page**

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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