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## 9231/11

May/June 2023

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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**1** Let  $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$ .

**(a)** Prove by mathematical induction that, for all positive integers  $n$ ,

$$2\mathbf{A}^n = \begin{pmatrix} 2 \times 3^n & 0 \\ 3^n - 1 & 2 \end{pmatrix}. \quad [5]$$

[illegible]

(b) Find, in terms of  $n$ , the inverse of  $\mathbf{A}^n$ . [2]

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2 The cubic equation  $x^3 + 4x^2 + 6x + 1 = 0$  has roots  $\alpha, \beta, \gamma$ .

(a) Find the value of  $\alpha^2 + \beta^2 + \gamma^2$ . [2]

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(b) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n ((\alpha + r)^2 + (\beta + r)^2 + (\gamma + r)^2) = n(n^2 + an + b),$$

where  $a$  and  $b$  are constants to be determined. [6]

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- 3 (a) Use the method of differences to find  $\sum_{r=1}^n \frac{1}{(kr+1)(kr-k+1)}$  in terms of  $n$  and  $k$ , where  $k$  is a positive constant. [4]

[illegible]

(b) Deduce the value of  $\sum_{r=1}^{\infty} \frac{1}{(kr+1)(kr-k+1)}$ . [1]

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(c) Find also  $\sum_{r=n}^{n^2} \frac{1}{(kr+1)(kr-k+1)}$  in terms of  $n$  and  $k$ . [2]

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- 4 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix}$ , where  $a, b, c$  are real constants and  $b \neq 0$ .

- (a) Show that  $\mathbf{M}$  does not represent a rotation about the origin. [2]

[illegible]

- (b) Find the equations of the invariant lines, through the origin, of the transformation in the  $x$ - $y$  plane represented by  $\mathbf{M}$ . [5]

[illegible]

It is given that  $\mathbf{M}$  represents the sequence of two transformations in the  $x$ - $y$  plane given by an enlargement, centre the origin, scale factor 5 followed by a shear,  $x$ -axis fixed, with  $(0, 1)$  mapped to  $(5, 1)$ .

- (c) Find  $\mathbf{M}$ . [3]

- (d) The triangle  $DEF$  in the  $x$ - $y$  plane is transformed by  $\mathbf{M}$  onto triangle  $PQR$ .

Given that the area of triangle  $DEF$  is  $12 \text{ cm}^2$ , find the area of triangle  $PQR$ . [2]

5 The curve  $C$  has polar equation  $r^2 = \frac{1}{\theta^2 + 1}$ , for  $0 \leq \theta \leq \pi$ .

(a) Sketch  $C$  and state the polar coordinates of the point of  $C$  furthest from the pole. [3]

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(b) Find the area of the region enclosed by  $C$ , the initial line, and the half-line  $\theta = \pi$ . [4]

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6 The curve  $C$  has equation  $y = \frac{x^2 + 2x - 15}{x - 2}$ .

(a) Find the equations of the asymptotes of  $C$ . [3]

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(b) Show that  $C$  has no stationary points. [3]

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- (c) Sketch  $C$ , stating the coordinates of the intersections with the axes.

[3]

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- (d) Sketch the curve with equation  $y = \left| \frac{x^2 - 2x - 15}{x - 2} \right|$ .

[2]

- (e) Find the set of values of  $x$  for which  $\left| \frac{2x^2 + 4x - 30}{x - 2} \right| < 15$ . [4]

[illegible]



7 The plane  $\Pi_1$  has equation  $r = -4\mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ .

(a) Obtain an equation of  $\Pi_1$  in the form  $px + qy + rz = d$ . [4]

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(b) The plane  $\Pi_2$  has equation  $\mathbf{r} \cdot (-5\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) = 4$ .

Find a vector equation of the line of intersection of  $\Pi_1$  and  $\Pi_2$ . [4]

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The line  $l$  passes through the point  $A$  with position vector  $a\mathbf{i} + a\mathbf{j} + (a-7)\mathbf{k}$  and is parallel to  $(1-b)\mathbf{i} + b\mathbf{j} + b\mathbf{k}$ , where  $a$  and  $b$  are positive constants.

- (c) Given that the perpendicular distance from  $A$  to  $\Pi_1$  is  $\sqrt{2}$ , find the value of  $a$ . [2]

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- (d) Given that the obtuse angle between  $l$  and  $\Pi_1$  is  $\frac{3}{4}\pi$ , find the exact value of  $b$ . [4]

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