



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/12

May/June 2023

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) Prove by mathematical induction that, for all positive integers n ,

$$2\mathbf{A}^n = \begin{pmatrix} 2 \times 3^n & 0 \\ 3^n - 1 & 2 \end{pmatrix}. \quad [5]$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find, in terms of n , the inverse of $\mathbf{A}^n$. [2]

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2 The cubic equation $x^3 + 4x^2 + 6x + 1 = 0$ has roots α, β, γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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(b) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n ((\alpha + r)^2 + (\beta + r)^2 + (\gamma + r)^2) = n(n^2 + an + b),$$

where a and b are constants to be determined. [6]

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- 3 (a) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k , where k is a positive constant. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

(b) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(kr+1)(kr-k+1)}$. [1]

(c) Find also $\sum_{r=n}^{n^2} \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k . [2]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix}$, where a, b, c are real constants and $b \neq 0$.

- (a) Show that $\mathbf{M}$ does not represent a rotation about the origin. [2]

- (b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{M}$. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

It is given that $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by an enlargement, centre the origin, scale factor 5 followed by a shear, x -axis fixed, with $(0, 1)$ mapped to $(5, 1)$.

- (c) Find $\mathbf{M}$. [3]

- (d) The triangle DEF in the x - y plane is transformed by $\mathbf{M}$ onto triangle PQR .

Given that the area of triangle DEF is 12 cm^2 , find the area of triangle PQR . [2]

5 The curve C has polar equation $r^2 = \frac{1}{\theta^2 + 1}$, for $0 \leq \theta \leq \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

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(b) Find the area of the region enclosed by C , the initial line, and the half-line $\theta = \pi$. [4]

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6 The curve C has equation $y = \frac{x^2 + 2x - 15}{x - 2}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Show that C has no stationary points. [3]

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- (c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 - 2x - 15}{x - 2} \right|$.

[2]

[illegible]

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7 The plane Π_1 has equation $r = -4\mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Obtain an equation of Π_1 in the form $px + qy + rz = d$. [4]

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(b) The plane Π_2 has equation $\mathbf{r} \cdot (-5\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) = 4$.

Find a vector equation of the line of intersection of Π_1 and Π_2 . [4]

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The line l passes through the point A with position vector $a\mathbf{i} + a\mathbf{j} + (a-7)\mathbf{k}$ and is parallel to $(1-b)\mathbf{i} + b\mathbf{j} + b\mathbf{k}$, where a and b are positive constants.

- (c) Given that the perpendicular distance from A to Π_1 is $\sqrt{2}$, find the value of a . [2]

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- (d) Given that the obtuse angle between l and Π_1 is $\frac{3}{4}\pi$, find the exact value of b . [4]

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Additional page

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