Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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0834557711

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Prove by mathematical induction that, for all positive integers n, $5^{3n} + 32^n - 33$ is divisible by 31. [6]

2 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(6r^2+6r-5) = an^3+bn^2+cn,$$

where a, b and c are integers to be determined. [2]

(b) Use the method of differences to find $\sum_{r=1}^{n}\frac{6r^2+6r-5}{r^2+r}$ in terms of n. [4]

(c) Find also $\sum_{r=n+1}^{2n} \frac{6r^2+6r-5}{r^2+r}$ in terms of n. [2]

3 The equation $x^4 - x^2 + 2x + 5 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\alpha^2, \beta^2, \gamma^2, \delta^2$ and state the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [4]

(b) Find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}+\frac{1}{\delta^2}$. [3]

(c) Find the value of $\alpha^4+\beta^4+\gamma^4+\delta^4$. [2]

4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where $0 < \theta < \pi$ and k is a non-zero constant. The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations, one of which is a shear.

(a) Describe fully the other transformation and state the order in which the transformations are applied. [3]

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(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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(c) Find, in terms of k, the value of $\tan\theta$ for which $\mathbf{M} - \mathbf{I}$ is singular. [5]

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(d) Given that $k = 2\sqrt{3}$ and $\theta = \frac{1}{3}\pi$, show that the invariant points of the transformation represented by **M** lie on the line $3y + \sqrt{3}x = 0$. [4]

5 **(a)** Show that the curve with Cartesian equation

$$x^2 - y^2 = a,$$

where a is a positive constant, has polar equation $r^2 = a\sec 2\theta$. [3]

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The curve C has polar equation $r^2 = a\sec 2\theta$, where a is a positive constant, for $0 \leqslant \theta < \frac{1}{4}\pi$.

(b) Sketch C and state the minimum distance of C from the pole. [3]

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(c) Find, in terms of a, the exact value of the area of the region enclosed by C, the initial line, and the half-line $\theta = \frac{1}{12}\pi$. [You may use any result from the list of formulae (MF19) without proof.] [4]

6 The points A, B, C have position vectors

$$\mathbf{i}+\mathbf{j}, \quad -\mathbf{i}+2\mathbf{j}+4\mathbf{k}, \quad -2\mathbf{i}+\mathbf{j}+3\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

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(b) Find the perpendicular distance from O to the plane ABC. [2]

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(c) Find a vector equation of the common perpendicular to the lines OC and AB. [8]

7 The curve C has equation $y = \dfrac{x^2 + 2x + 1}{x - 3}$.

(a) Find the equations of the asymptotes of C. [3]

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(b) Find the coordinates of the turning points on C. [3]

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(c) Sketch C. [3]

(d) Sketch the curves with equations $y = \left| \frac{x^2 + 2x + 1}{x - 3} \right|$ and $y^2 = \frac{x^2 + 2x + 1}{x - 3}$ on a single diagram, clearly identifying each curve. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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