



CANDIDATE
NAME

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CENTRE
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CANDIDATE
NUMBER

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9231/23

May/June 2023

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 (a)** Find the Maclaurin series for $\sin^{-1}x$ up to and including the term in x^3 . [5]

[illegible]

- (b)** Deduce an approximation to $\int_0^{\frac{1}{5}} \frac{1}{\sqrt{1-u^2}} du$, giving your answer as a fraction. [1]

[illegible]

2 The variables x and y are related by the differential equation

$$6\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x = t^2 + 10t + 13.$$

(a) Find the general solution for x in terms of t . [6]

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(b) State an approximate solution for large positive values of t . [1]

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- 3** By considering the binomial expansions of $\left(z + \frac{1}{z}\right)^4$ and $\left(z - \frac{1}{z}\right)^4$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cot^4 \theta = \frac{\cos 4\theta + a \cos 2\theta + b}{\cos 4\theta - a \cos 2\theta + b},$$

where a and b are integers to be determined.

[7]

[illegible]

4 The curve C has equation

$$4y^3 + (x+y)^6 = 109.$$

(a) Show that, at the point $(-4, 3)$ on C , $\frac{dy}{dx} = \frac{1}{17}$. [3]

[illegible]

- (b)** Find the value of $\frac{d^2y}{dx^2}$ at the point $(-4, 3)$. [5]

[illegible]

- 5 (a) Starting from the definitions of $\cosh$ and $\sinh$ in terms of exponentials, prove that

$$2 \cosh^2 x = \cosh 2x + 1. \quad [3]$$

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- (b) Find the solution of the differential equation

$$\frac{dy}{dx} + 2y \tanh x = 1$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. [8]

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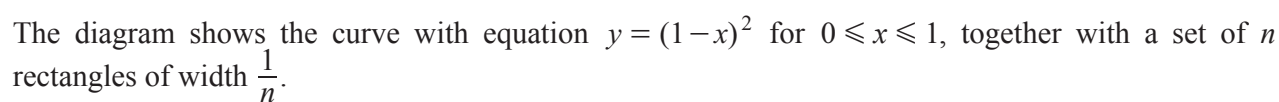
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- $$U_n = \frac{2n^2 + 3n + 1}{6n^2}. \quad [5]$$

[illegible]

- (b)** Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (1-x)^2 dx$. [4]

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- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

[illegible]

7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{4}{3}} (1+x^2)^{\frac{1}{2}n} dx$.

- (a) Find the exact value of I_{-1} giving your answer in the form $\ln a$, where a is an integer to be determined. [2]

[illegible]

- (b)** By considering $\frac{d}{dx} \left(x(1+x^2)^{\frac{1}{2}n} \right)$, or otherwise, show that

$$(n+1)I_n = nI_{n-2} + \frac{4}{3}\left(\frac{5}{3}\right)^n. \quad [5]$$

[illegible]

- (c) A curve has equation $y = x^2$, for $0 \leq x \leq \frac{2}{3}$. The arc length of the curve is denoted by s .

Use the substitution $u = 2x$ to show that $s = \frac{1}{2}I_1$ and find the exact value of s . [4]

[illegible]

8 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & -6a & 2a+2 \\ 0 & 1-a & 0 \\ 0 & 2-a & -1 \end{pmatrix}$$

where a is a constant with $a \neq 0$ and $a \neq 1$.

- (a) Show that the equation $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ has a unique solution and interpret this situation geometrically.

[3]

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- (b) Show that the eigenvalues of $\mathbf{A}$ are a , $1-a$ and -1 .

[2]

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(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^4$ in terms of $\mathbf{A}$ and a . [3]

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[illegible]

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