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9231/43

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}) & 1 \leq x \leq 27, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the cumulative distribution function of X . [3]

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The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

- (b) Find the probability density function of Y . [3]

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(c) Find the exact value of the median of Y .

[2]

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- 2 Shane is studying the lengths of the tails of male red kangaroos. He takes a random sample of 14 male red kangaroos and measures the length of the tail, x m, for each kangaroo. He then calculates a 90% confidence interval for the population mean tail length, μ m, of male red kangaroos. He assumes that the tail lengths are normally distributed and finds that $1.11 \leq \mu \leq 1.14$.

Find the values of $\sum x$ and $\sum x^2$ for this sample.

[6]

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- 3** A large number of students took two test papers in mathematics. The teacher believes that the marks obtained in Paper 1 will be higher than the marks obtained in Paper 2. She chooses a random sample of 9 students and compares their marks. The marks are shown in the table.

Student	A	B	C	D	E	F	G	H	I
Paper 1	46	73	55	64	86	42	66	68	60
Paper 2	41	66	61	63	90	40	58	42	70

- (a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the teacher's belief. [7]

[illegible]

- (b) State an assumption that you have made in carrying out the test in part (a). [1]

- 4 An inspector is checking the lengths of metal rods produced by two machines, X and Y . These rods should be of the same length, but the inspector suspects that those made by machine X are shorter, on average, than those made by machine Y . The inspector chooses a random sample of 80 rods made by machine X and a random sample of 60 rods made by machine Y . The lengths of these rods are x cm and y cm respectively. Her results are summarised as follows.

$$\Sigma x = 164.0 \quad \Sigma x^2 = 338.1 \quad \Sigma y = 124.8 \quad \Sigma y^2 = 261.1$$

- (a) Test at the 10% significance level whether the data supports the inspector's suspicion. [8]

[illegible]

- (b) Give a reason why it is not necessary to make any assumption about the distributions of the lengths of the rods. [1]

- 5 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = k(1 + 3t + 4t^2),$$

where k is a constant.

- (a) Show that $E(X) = \frac{11}{8}$. [3]

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The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{1}{3}t^2(1 + 2t).$$

The random variables X and Y are independent and $Z = X + Y$.

- (b) Find the probability generating function of Z , expressing your answer as a polynomial in t . [2]

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(c) Use your answer to part (b) to find the value of $\text{Var}(Z)$.

[3]

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(d) Write down the most probable value of Z .

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- 6 A scientist is investigating whether the ability to remember depends on age. A random sample of 150 students in different age groups is chosen. Each student is shown a set of 20 objects for thirty seconds and then asked to list as many as they can remember. The students are graded A or B according to how many objects they remembered correctly: grade A for 16 or more correct and grade B for fewer than 16 correct. The results are shown in the table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade A	25	16	19
Grade B	28	45	17

- (a) Carry out a χ^2 -test at the 2.5% significance level to test whether grade is independent of age of student. [7]

[illegible]

The scientist decides instead to use three grades: grade *A* for 16 or more correct, grade *B* for 10 to 15 correct and grade *C* for fewer than 10 correct. The results are shown in the following table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade <i>A</i>	25	16	19
Grade <i>B</i>	12	27	11
Grade <i>C</i>	16	18	6

With this second set of data, the test statistic is calculated as 10.91.

- (b) Complete the χ^2 -test at the 2.5% significance level for this second set of data. [2]

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- (c) State, with a reason, whether you would prefer to use the result from part (a) or part (b) to investigate whether the ability to remember depends on age. [1]

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