



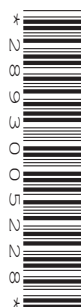
# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.





1 The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where  $k$  is a real constant.

(a) Show that  $\mathbf{A}$  is non-singular.

[3]

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(b) Given that  $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$ , find the value of  $k$ .

[2]

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(a) Find a cubic equation whose roots are  $\alpha^2 + 1, \beta^2 + 1, \gamma^2 + 1$ . [3]

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- (b) Find the value of  $(\alpha^2 + 1)^2 + (\beta^2 + 1)^2 + (\gamma^2 + 1)^2$ . [2]

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- (c) Find the value of  $(\alpha^2 + 1)^3 + (\beta^2 + 1)^3 + (\gamma^2 + 1)^3$ . [2]

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3 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$ .

(a) The matrix  $\mathbf{M}$  represents a sequence of two geometrical transformations in the  $x$ - $y$  plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Find the equations of the invariant lines, through the origin, of the transformation represented by  $\mathbf{M}$ . [5]

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The triangle  $DEF$  in the  $x$ - $y$  plane is transformed by  $\mathbf{M}$  onto triangle  $PQR$ .

- (c) Given that the area of triangle  $PQR$  is  $35 \text{ cm}^2$ , find the area of triangle  $DEF$ . [2]

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$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1).$$

[5]

[illegible]



5

show that  $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$ . [6]

[illegible]

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- 5 The lines  $l_1$  and  $l_2$  have equations  $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$  and  $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$  respectively.

- (a) Find the shortest distance between  $l_1$  and  $l_2$ . [5]

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The plane  $\Pi_1$  contains  $l_1$  and is parallel to  $l_2$ .

- (b) Obtain an equation of  $\Pi_1$  in the form  $px + qy + rz = s$ . [2]

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Obtain an equation of  $\Pi_2$  in the form  $ax + by + cz = d$ . [3]

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**(a)** Show that  $C$  has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

[illegible][illegible]



(c) Sketch  $C$ , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch  $y^2 = \frac{x+1}{x^2+3}$ , stating the coordinates of the stationary points and the intersections with the axes. [4]



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[4]

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[6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]



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