



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- $$\sum_{r=1}^n (2-3r)(5-3r) = an^3 + bn^2 + cn,$$

[3]

[illegible]



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- (a) Find a cubic equation whose roots are $\alpha^3 - 1$, $\beta^3 - 1$, $\gamma^3 - 1$.

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(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$.

[2]

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(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$.

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- (a) Prove by induction that $u_n = 6^n - 1$ for all positive integers n . [5]

[illegible]

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4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$, where $0 < \theta < 2\pi$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) Find the value of θ for which the transformation represented by $\mathbf{M}$ has a line of invariant points. [7]

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- (a) Sketch C.

[2]

- (b)** Find the area of the region bounded by C and the initial line, giving your answer in the form $(p\pi^2 + q\pi + r)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

[illegible]



- (c) Show that, at the point of C furthest from the initial line,

$$\theta \cos \theta + \left(\frac{1}{8}\theta + 1\right) \sin \theta = 0$$

and verify that this equation has a root between 5 and 5.05. [5]





6 The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD . [4]

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[7]

[illegible]

DO NOT WRITE IN THIS MARGIN

[2]

[illegible]

[4]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch the curve with equation $y = \left| \frac{2x^2 - 5x}{2x^2 - 7x - 4} \right|$.

[1]



[5]

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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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