



# Cambridge International AS & A Level

CANDIDATE  
NAMECENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

## FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

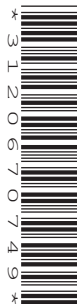
You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

[5]

[illegible]


$$\tanh^2 t + \operatorname{sech}^2 t = 1. \quad [3]$$
$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leq t \leq 1.$$

Find the length of  $C$ . [5]

$$9y^2 - 3 \sinh^{-1}(xy) = 1 - 3 \ln 3.$$

- (a) Show that, at the point  $(4, \frac{1}{3})$  on  $C$ ,  $\frac{dy}{dx} = -\frac{1}{2}$ . [4]

[illegible]



[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

4

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 2t^2 + t - 1,$$

given that, when  $t = 0$ ,  $x = \frac{dx}{dt} = 0$ . [10]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



5

[6]

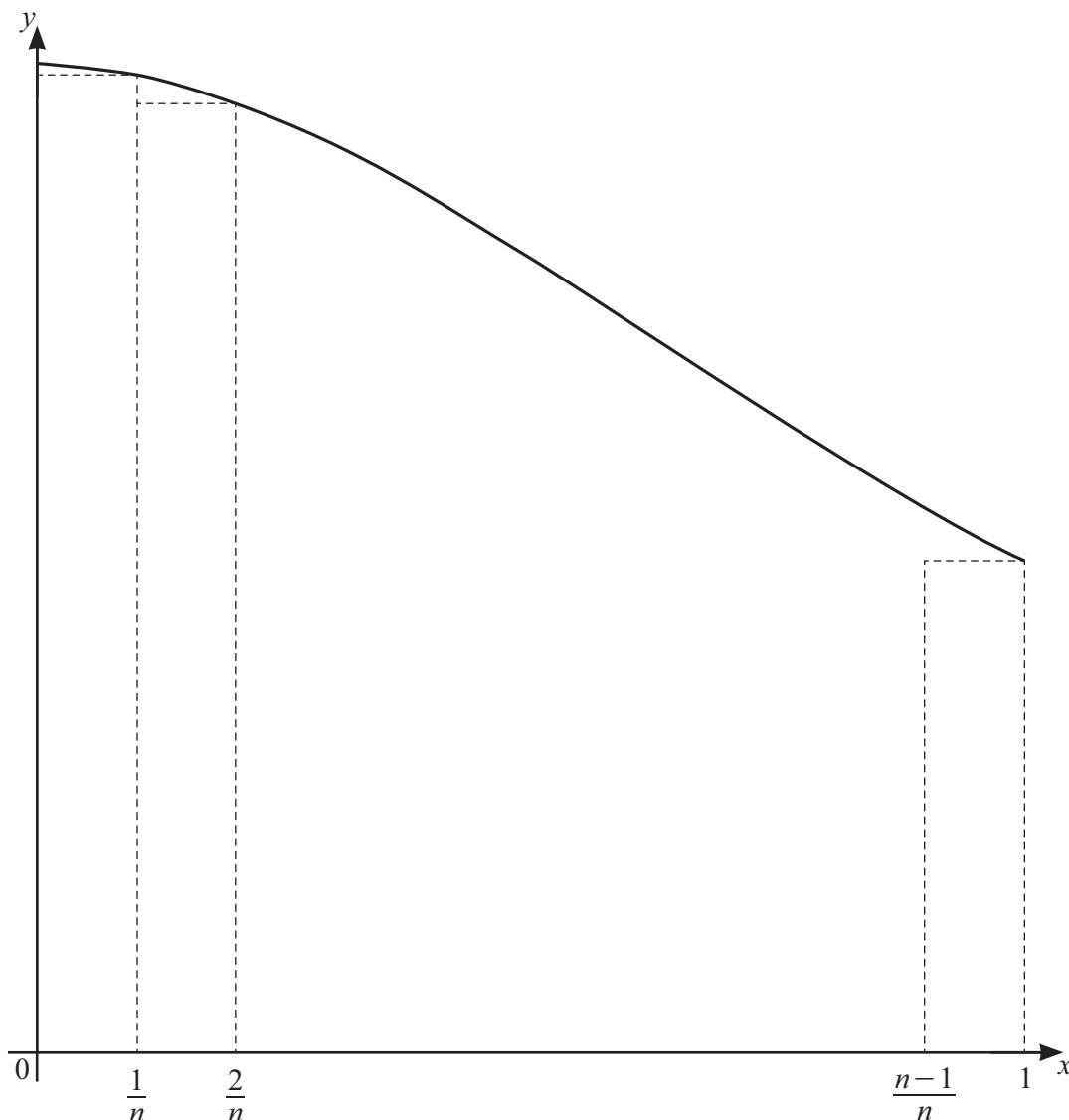
[illegible]


$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

[4]

[illegible]

6



The diagram shows the curve with equation  $y = \frac{1}{x^2 + 1}$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4}\pi. \quad [5]$$

.....

.....

.....

.....

.....

.....





- (b) Use a similar method to find a lower bound for  $\sum_{r=1}^n \frac{n}{n^2 + r^2}$ . Give your answer in terms of  $n$  and  $\pi$ . [4]

- (c) Deduce the exact value of  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$ . [1]



given that  $y = 0$  when  $x = 0$ . Give your answer in an exact form.

[9]

[illegible]





- 8 (a) Find the values of  $a$  for which the system of equations

$$\frac{3}{2}x + 3y + 8z = 1,$$

$$ax + 3y + 4z = 2,$$

$$ay - z = 3,$$

does not have a unique solution.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

- (b) Given that  $\mathbf{B} = \mathbf{A}^{-1}$ , use the characteristic equation of  $\mathbf{A}$  to show that  $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$ , where  $p$  and  $q$  are constants to be determined. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





[7]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, providing a clear guide for letter formation and alignment. There are no margins, text, or other markings present.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.