

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/13**

Paper 1 Further Pure Mathematics 1

May/June 2026**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **20** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$b = -(\alpha + \beta + \gamma) = -5$	B1	
	$-19 = 5^2 - 2c$	M1	Uses formula for sum of squares.
	$c = 22$	A1	
	EITHER: $11 = \frac{22}{-d}$	(M1)	Uses $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$. Using correct coefficients.
	$d = -2$	A1FT)	Follow through on their c .
	OR: Alternative method for the last 2 marks		
	$S_2 - 5S_1 + 22S_0 + dS_{-1} = 0$ $-19 - 5(5) + 22(3) + 11d = 0$ $11d = 19 + 5(5) - 22(3)$	(M1)	Uses $S_2 - 5S_1 + 22S_0 + dS_{-1} = 0$ and $S_0 = 3$
	$d = -2$	A1FT)	Follow through on their c .
	Available marks	5	
1(b)	$S_3 + bS_2 + cS_1 + 3d = 0$	M1	Uses original equation or formula for sum of cubes, allow one arithmetic error. Allow FT of their b , c , and d .
	$S_3 = -b(-19) - c(5) - 3(-2) = -199$	A1FT	Allow FT of their b , c , and d .
		2	

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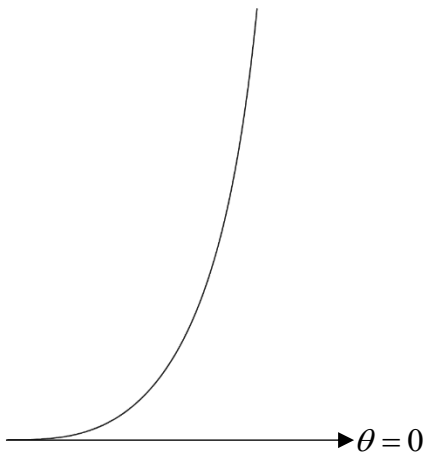
Question	Answer	Marks	Guidance
2(a)	Shear, rotation.	B1	Both words.
	Correct order.	B1	Two transformations in the correct order with at least one correct and not both the same transformation.
	[x-axis fixed], with (0,1) mapped to (2,1).	B1	For the description of the shear. Ignore any comment relating to scale or shear factor
	$\frac{1}{4}\pi$ [anticlockwise] about the origin.	B1	For rotation. Allow 45°
		4	
2(b)	$\mathbf{M} = \frac{1}{2}\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$	B1	OE and must have a scalar multiple of $\frac{1}{2}\sqrt{2}$.
	$\frac{1}{2}\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\sqrt{2}\begin{pmatrix} x+y \\ x+3y \end{pmatrix}$	B1FT	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$. Follow through for their M
	$\frac{1}{2}\sqrt{2}(x+3mx) = \frac{m}{2}\sqrt{2}(x+mx)$	M1	Uses $y = mx$ and $Y = mX$.
	$x+3mx = m(x+mx) \Rightarrow 1+3m = m+m^2 \Rightarrow m^2 - 2m - 1 = 0$	A1	OE.
	$y = (1+\sqrt{2})x$ and $y = (1-\sqrt{2})x$	A1	If ambiguity over invariant points instead of invariant lines, then M0 but SCB1 for the correct equations.
		5	

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Question	Answer	Marks	Guidance
3	$\mathbf{P}^{-1} = -\frac{1}{6} \begin{pmatrix} 3 & -3 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{7}{6} & -\frac{5}{6} \end{pmatrix}$	B1	
	$\mathbf{PDP}^{-1} = \begin{pmatrix} 5 & 3 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{7}{6} & -\frac{5}{6} \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{7}{6} & -\frac{5}{6} \end{pmatrix}$	M1	Base case, multiplies PD or $\mathbf{DP}^{-1} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{7}{6} & -\frac{5}{6} \end{pmatrix}$.
	$\mathbf{PDP}^{-1} = \begin{pmatrix} 8 & -5 \\ 7 & -4 \end{pmatrix} = \mathbf{A}$	A1	Reference to base case
	Assume that it is true for $n = k$, so $\mathbf{A}^k = \mathbf{PD}^k \mathbf{P}^{-1}$.	B1	States inductive hypothesis.
	Then $\mathbf{A}^{k+1} = \mathbf{A}^k \mathbf{A} = \mathbf{PD}^k \mathbf{P}^{-1} \mathbf{PDP}^{-1} = \mathbf{PD}^{k+1} \mathbf{P}^{-1}$	M1A1	Multiplies $\mathbf{A}^k$ with $\mathbf{A}$ and substitutes $\mathbf{A}^k = \mathbf{PD}^k \mathbf{P}^{-1}$.
	So $\mathbf{A}^n = \mathbf{PD}^n \mathbf{P}^{-1}$ is true for all positive [integers] n [by induction].	A1	States conclusion.
		7	

Question	Answer	Marks	Guidance
4(a)	$16r^4 + 32r^3 + 24r^2 + 8r + 1 - (16r^4 - 32r^3 + 24r^2 - 8r + 1) = 64r^3 + 16r$	M1	Some evidence of expansion or evidence of using difference of two squares. Allow one error overall.
		A1	Correct answer with like terms collected.
	$r = 1 \quad 3^4 - 1^4$ $r = 2 \quad 5^4 - 3^4$...	M1	Write enough correct <u>rows</u> of terms so that all that are left in the answer appear plus one pair that cancel.
	$r = n \quad (2n+1)^4 - (2n-1)^4$ $\left[64 \sum_{r=1}^n r^3 + 16 \sum_{r=1}^n r \right] (2n+1)^4 - 1$	A1	All necessary <u>rows</u> of terms to demonstrate the cancellation. Either two at the beginning and one at the end or one at the beginning and two at the end.
	$\Rightarrow 64 \sum_{r=1}^n r^3 = 16n^4 + 32n^3 + 24n^2 + 8n - 8n(n+1)$	M1	Applies $\sum_{r=1}^n r = \frac{1}{2} n(n+1)$.
	$\Rightarrow 64 \sum_{r=1}^n r^3 = 16n^2 (n^2 + 2n + 1) \Rightarrow \sum_{r=1}^n r^3 = \frac{1}{4} n^2 (n+1)^2$	A1	AG. Must see at least one intermediate step of working before the answer given.
		6	

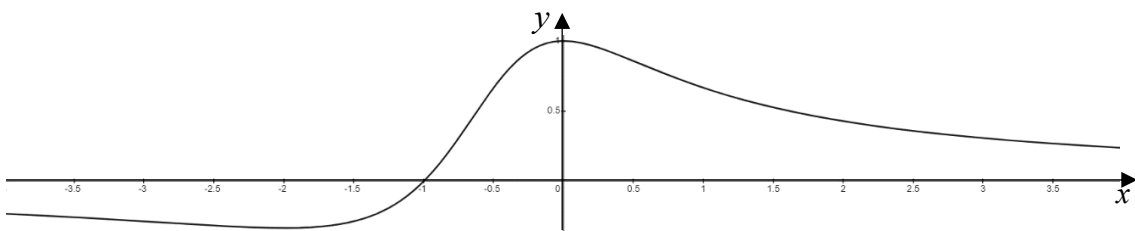
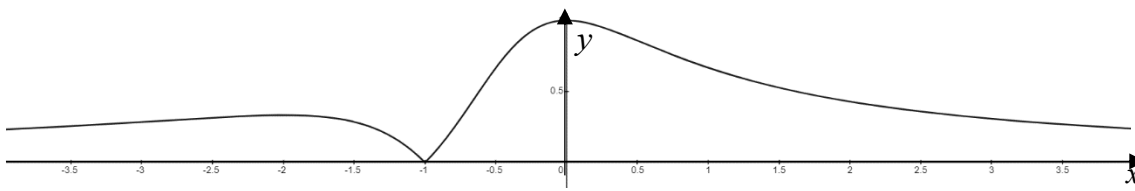
Question	Answer	Marks	Guidance
4(b)	EITHER: $[S_{2n}] = \sum_{r=1}^{2n} r^3 - 2 \sum_{r=1}^n (2r)^3$	(*M1)	Relates with sum of cubes.
	$[S_{2n}] = \frac{1}{4}(2n)^2(2n+1)^2 - \frac{16}{4}n^2(n+1)^2$	DM1	Applies $\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$.
	$S_{2n} = n^2((2n+1)^2 - 4(n+1)^2) = n^2(-4n-3)$	A1)	Accept $-n^2(4n+3)$
	OR: Alternative method		
	$[S_{2n}] = (1^3 + 3^3 + \dots + (2n-1)^3) - (2^3 + 4^3 + \dots + (2n)^3)$	(*M1)	Relates with difference of cubes
	$[S_{2n}] = -12 \sum_{r=1}^n r^2 + 6 \sum_{r=1}^n r - n$ $[S_{2n}] = \frac{-12n}{6}(n+1)(2n+1) + \frac{6}{2}n(n+1) - n$	DM1	Applies $\sum_{r=1}^n r^2 = \frac{n}{6}(n+1)(2n+1)$ and $\sum_{r=1}^n r = \frac{n}{2}(n+1)$
	$S_{2n} = -4n^3 - 6n^2 - 2n + 3n^2 + 3n - n = n^2(-4n-3)$	A1)	Accept $-n^2(4n+3)$
	Available marks	3	

Question	Answer	Marks	Guidance
5(a)		B1	Initial line drawn and correct parabolic shape (not vertical at furthest point), starts from the pole.
	[Maximum distance of C from the pole is] $2\left(\frac{\pi}{3}\right)^{\frac{1}{2}}$.	B1	Accept AWRT 2.05
		2	
5(b)	$\frac{1}{2} \int_0^{\frac{1}{3}\pi} \theta \sec^2 \theta \, d\theta$	M1	Uses $\frac{1}{2} \int r^2 \, d\theta$ with correct limits. Condone omission of $d\theta$.
	$= \frac{1}{2} \left[\theta \tan \theta \right]_0^{\frac{1}{3}\pi} - \frac{1}{2} \int_0^{\frac{1}{3}\pi} \tan \theta \, d\theta$	M1	Integrates $\theta \sec^2 \theta$ by parts.
		A1	Allow omission of the half and limits for these marks.
	$= \frac{1}{2} \left[\theta \tan \theta - \ln \sec \theta \right]_0^{\frac{1}{3}\pi}$	A1	OE. Must have the half and the limits for this mark.
	$= \frac{1}{2} \left(\frac{1}{3} \pi \sqrt{3} - \ln 2 \right)$	A1	OE, must be exact. ISW
		5	

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Question	Answer	Marks	Guidance
5(c)	$r^2 \cos^2 \theta = \theta \Rightarrow x^2 = \theta$	M1	Uses $x = r \cos \theta$.
	$x^2 = \tan^{-1} \frac{y}{x}$ $\tan(x^2) = \frac{y}{x}$	M1	Applies $\tan \theta = \frac{y}{x}$
	$y = x \tan(x^2)$	A1	AG. Must see an intermediary step in the working before the answer given
		3	
5(d)	$\frac{1}{2} \left(2 \left(\frac{\pi}{3} \right)^{\frac{1}{2}} \cos \frac{1}{3} \pi \right) \left(2 \left(\frac{\pi}{3} \right)^{\frac{1}{2}} \sin \frac{1}{3} \pi \right) = \frac{1}{6} \pi \sqrt{3}$	B1	Area of triangle.
	$\frac{1}{6} \pi \sqrt{3} - \frac{1}{2} \left(\frac{1}{3} \pi \sqrt{3} - \ln 2 \right)$	M1	Area of triangle minus <i>their</i> answer to (b) (<i>their</i> answer in (b) can be a decimal).
	$\frac{1}{2} \ln 2$	A1	OE
		3	

Question	Answer	Marks	Guidance
6(a)	$x^2 + x + 1 = 0$ has no real roots as discriminant is $[1^2 - 4 =] -3$ and this is less than zero, hence shown.	B1	The discriminant needs be evaluated, compared and concluded. Or shows that the expression is always positive (i.e. completing the square) and concludes.
	$y = 0$	B1	Horizontal asymptote.
		2	

Question	Answer	Marks	Guidance
6(b)	$\frac{dy}{dx} = \frac{(x^2 + x + 1)(1) - (x + 1)(2x + 1)}{(x^2 + x + 1)^2}$	M1	Finds $\frac{dy}{dx}$. Allow one arithmetic error for this mark.
	$x^2 + 2x = 0$	M1	Sets equal to 0, forms equation and attempts to solve.
	$(0,1)$ and $(-2, -\frac{1}{3})$	A1	Condone clearly paired x and y .
		3	
6(c)		B1	Axes and approach to asymptote.
		B1	Correct shape and position, and maximum apparently on the axis
	$(0,1)$ and $(-1,0)$	B1	States coordinates of just the two intersections with axes, may be seen on their diagram.
		3	
6(d)		B1FT	FT provided sketch in part (c) has both positive and negative y values
		B1	Correct shape at $(-1,0)$ and must have a correct graph in part (c)
		2	

Question	Answer	Marks	Guidance
6(e)	$\frac{x+1}{x^2+x+1} = \frac{1}{3}$ or $\frac{x+1}{x^2+x+1} < \frac{1}{3}$	M1	Writing down the equation or inequality
	$\frac{x+1}{x^2+x+1} = -\frac{1}{3}$ or $\frac{x+1}{x^2+x+1} > -\frac{1}{3}$	M1	Writing down the equation or inequality
	EITHER: $x < 1 - \sqrt{3}, x > 1 + \sqrt{3}$	(A1	Writing both inequalities ISW
	$x \neq -2$	A1)	ISW
	OR: Alternative for the final 2 marks		
	$x < -2, -2 < x < 1 - \sqrt{3}, x > 1 + \sqrt{3}$	(A2,0)	All three regions correct.
	Available marks	4	

Question	Answer	Marks	Guidance
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ -3 & 1 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$	M1 A1	Finds normal to the plane Π . For the method mark allow one arithmetic error in calculating the cross product but must be using the correct direction vectors.
	$(0) + (1) - 2(2) = -3 \Rightarrow x + y - 2z = -3$	M1	Substitutes point using their cross product but must be from using two correct direction vectors.
		A1	Answer must be in the form $ax + by + cz = d$.
		4	
7(b)	$\frac{1}{\sqrt{6}} \left -3 - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6+6\theta \\ 1+\theta \\ 2+5\theta \end{pmatrix} \right = \frac{ 3\theta-6 }{\sqrt{6}}$	*M1 A1	Forms perpendicular distance of P from plane Π in terms of θ . For only the method mark allow follow through of their d from part (a).
	$ 3\theta-6 < 6$	DM1	Forms and solves linear inequality in θ .
	$0 < \theta < 4$	A1	
		4	

Question	Answer	Marks	Guidance
7(c)	$\overrightarrow{OQ} = \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \overrightarrow{OR} = \begin{pmatrix} 1-3\mu \\ \mu \\ 2-\mu \end{pmatrix} \Rightarrow \overrightarrow{QR} = \begin{pmatrix} 1-3\mu \\ \mu-\lambda \\ 2-\mu-2\lambda \end{pmatrix}$	M1	Allow one arithmetic error in calculating one element.
		A1	Finds $\overrightarrow{QR}$ or direction of common perpendicular.
	$\begin{pmatrix} 1-3\mu \\ \mu-\lambda \\ 2-\mu-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = 0$	M1	Uses that dot product of $\overrightarrow{QR}$ with line direction is zero, or, alternatively, $\overrightarrow{QR}$ is a multiple of the common perpendicular $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, the multiple is not 1.
	$\mu + 5\lambda = 4$	A1	Deduces one equation.
	$\begin{pmatrix} 1-3\mu \\ \mu-\lambda \\ 2-\mu-2\lambda \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} = 0 \Rightarrow 11\mu + \lambda = 5$	M1	Deduces second equation for their $\overrightarrow{QR}$
	$\mu = \frac{7}{18} \Rightarrow \overrightarrow{OR} = \begin{pmatrix} -\frac{1}{6} \\ \frac{7}{18} \\ \frac{29}{18} \end{pmatrix} = \frac{1}{18} \begin{pmatrix} -3 \\ 7 \\ 29 \end{pmatrix}$ $\overrightarrow{QR} = \begin{pmatrix} -\frac{1}{6} \\ -\frac{1}{3} \\ \frac{1}{6} \end{pmatrix}$	M1 A1	Solves for μ and substitutes into $\overrightarrow{OR}$ and $\overrightarrow{QR}$ or $\lambda = \frac{13}{18} \Rightarrow \overrightarrow{OQ} = \frac{13}{18} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -\frac{1}{6} \\ -\frac{1}{3} \\ \frac{1}{6} \end{pmatrix}$

Question	Answer	Marks	Guidance
7(c)	$\mathbf{r} = \frac{1}{18} \begin{pmatrix} -3 \\ 7 \\ 29 \end{pmatrix} + k \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	B1 FT	OE. FT using their common perpendicular. Answer must be in equation form.
		8	