



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2026

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the roots of the equation $w^5 + 32 = 0$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



- 2** A curve has equation $y = \sqrt{x}$, for $0 \leq x \leq 2$. Find, in terms of π , the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [5]

[illegible]



3 Let $I_n = \int_0^a (\ln(x+1))^n dx$, where n is a non-negative integer and $a > 0$.

(a) By considering the derivative of $(x+1)(\ln(x+1))^n$, or otherwise, show that for $n \geq 1$

$$I_n = (a+1)(\ln(a+1))^n - nI_{n-1}.$$

[3]





(b) Find I_2 in terms of a .

[3]

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4 Find the particular solution of the differential equation

$$49\frac{d^2x}{dt^2} + 14\frac{dx}{dt} + x = 196e^{\frac{1}{7}t},$$

given that, when $t = 0$, $x = 7$ and $\frac{dx}{dt} = 0$.

[10]

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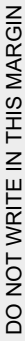
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- 5 (a) Starting from the definitions of sech and $\tanh$ in terms of exponentials, prove that

$$1 - \operatorname{sech}^2 t = \tanh^2 t.$$

[3]

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It is given that

$$x = t \sinh t \quad \text{and} \quad y = t \cosh t.$$

- (b) Show that $\frac{dy}{dx} = \frac{t \tanh t + 1}{t + \tanh t}$.

[3]

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(c) Show that $\frac{d^2y}{dx^2} = \frac{(t^2 - 2)\operatorname{sech}^n t}{(t + \tanh t)^n}$, where n is an integer to be determined.

[6]

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6 (a) Show that an appropriate integrating factor for

$$(1-x^2)\frac{dy}{dx} + y = (1-x^2)\sqrt{1-x}$$

is $\sqrt{\frac{1+x}{1-x}}$.

[4]

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(b) Hence find the solution of the differential equation

$$(1-x^2)\frac{dy}{dx} + y = (1-x^2)\sqrt{1-x}$$

for which $y = 1$ when $x = 0$.

[6]

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- 7 (a) Use the substitution $u = 1 - x^2$ to find $\int \frac{x}{\sqrt{1-x^2}} dx$. [3]

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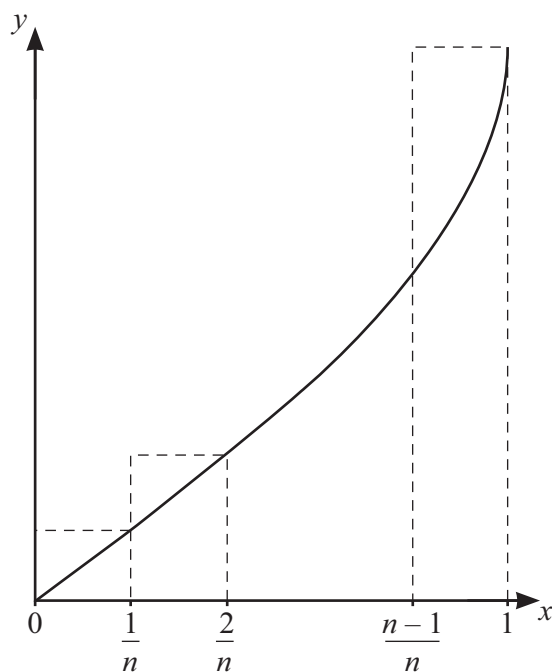
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The diagram shows the curve with equation for $y = \sin^{-1}x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right) \geq \frac{1}{2}\pi - 1. \quad [5]$$

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- (c) Use a similar method to find, in terms of n , an upper bound for $\sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [3]

- (d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \sin^{-1}\left(\frac{r}{n}\right)$. [1]



- 8 (a) The constants a and b are such that the system of equations

$$6x - 12y - 12z = 1,$$

$$ax + 4y + z = 2,$$

$$2x + by - 8z = 3,$$

does not have a unique solution. Find a in terms of b .

[3]

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- (b) Given that $a = -\frac{1}{4}$ and $b = -32$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically.

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -12 & -12 \\ -1 & 4 & 1 \\ 2 & -8 & -8 \end{pmatrix},$$

- (c) Show that $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

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- (d) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [7]

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Additional page

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