



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/11

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$, where a and b are positive constants.

- (a)** The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the x - y plane is transformed by $\mathbf{M}$ onto parallelogram $OPQR$.

- (b) Find, in terms of a and b , the matrix which transforms parallelogram $OPQR$ onto the unit square. [2]

[illegible]

It is given that the area of $OPQR$ is 2 cm^2 and that the line $x+3y=0$ is invariant under the transformation represented by $\mathbf{M}$.

(c) Find the values of a and b .

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a) Use standard results from the List of Formulae (MF19) to show that

$$\sum_{r=1}^n (7r+1)(7r+8) = an^3 + bn^2 + cn,$$

where a , b and c are constants to be determined.

[3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (b) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(7r+1)(7r+8)}$ in terms of n . [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(7r+1)(7r+8)}$. [1]

3 The cubic equation $x^3 + cx + 1 = 0$, where c is a constant, has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^3, \beta^3, \gamma^3$. [3]

[illegible]

(b) Show that $\alpha^6 + \beta^6 + \gamma^6 = 3 - 2c^3$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) Find the real value of c for which the matrix $\begin{pmatrix} 1 & \alpha^3 & \beta^3 \\ \alpha^3 & 1 & \gamma^3 \\ \beta^3 & \gamma^3 & 1 \end{pmatrix}$ is singular. [5]

[illegible]

4 The points A, B, C have position vectors

$$-\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \quad -2\mathbf{i} - \mathbf{j}, \quad 2\mathbf{i} + 2\mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Find the perpendicular distance from O to the plane ABC . [2]

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- (c) Find the acute angle between the planes OAB and ABC . [4]

[illegible]

5 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \sin x) = (-1)^{n-1} (x \cos x + (2n-1) \sin x). \quad [7]$$

[illegible]

6 The curve C has equation $y = \frac{x^2 + x - 1}{x - 1}$.

(a) Find the equations of the asymptotes of C .

[3]

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(b) Show that there is no point on C for which $1 < y < 5$.

[4]

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- (c) Find the coordinates of the intersections of C with the axes, and sketch C . [3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 + x - 1}{x - 1} \right|$. [2]

7 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{5}{2}} = 4xy(x^2 - y^2)$$

has polar equation $r = \sin 4\theta$.

[4]

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The curve C has polar equation $r = \sin 4\theta$, for $0 \leq \theta \leq \frac{1}{4}\pi$.

- (b)** Sketch C and state the equation of the line of symmetry. [3]

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- (c) Find the exact value of the area of the region enclosed by C . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (d) Using the identity $\sin 4\theta \equiv 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$, find the maximum distance of C from the line $\theta = \frac{1}{2}\pi$. Give your answer correct to 2 decimal places. [6]

[illegible]

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