Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4597910349

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 **(a)** By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]

(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

2 The variables x and y are related by the differential equation

$$9\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6\frac{\mathrm{d}y}{\mathrm{d}x}+y=3x^2+30x.$$

(a) Find the general solution for y in terms of x. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) State an approximate solution for large positive values of x. [1]

.....

.....

.....

.....

3 **(a)** Show that the system of equations

$$\begin{aligned} x-2y-4z &= 1, \\ x-2y+kz &= 1, \\ -x+2y+2z &= 1, \end{aligned}$$

where k is a constant, does not have a unique solution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Given instead that $k=-2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) For the case where $k\neq-2$ and $k\neq-4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

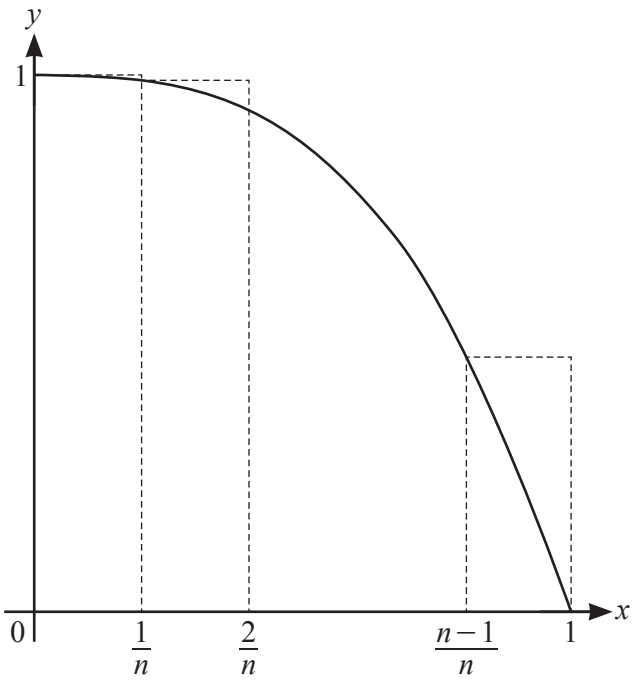
.....

.....

.....

.....

4



The diagram shows the curve with equation $y = 1 - x^3$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3)\,\mathrm{d}x \leqslant \frac{3n^2 + 2n - 1}{4n^2}. \qquad [4]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use a similar method to find, in terms of n, a lower bound for $\int_0^1 (1-x^3)\,\mathrm{d}x$. [4]

5 It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t, show that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{1}{\sqrt{1-t^2}}$. [4]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer. [5]

6 **(a)** Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [5]

(b) Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} + y\cot\theta = \sin^3\theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$. [6]

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

.....

.....

.....

.....

.....

(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The 3×3 matrix **A** has distinct eigenvalues b, -1, 1 with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

(c) Find **A** in terms of b. [4]

8 **(a)** Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

(b) Starting from the definitions of coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$

The curve C has equation $y = \ln\coth\left(\frac{1}{2}x\right)$ for $x > 0$.

(c) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosech} x$. [3]

(d) It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....