Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

FURTHER MATHEMATICS **9231/22**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (RW/SG) 187470/2

1 Find the Maclaurin's series for $\tan\left(x+\frac{1}{4}\pi\right)$ up to and including the term in x^2. [5]

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2 A curve has equation $y = \cosh x$, for $0 \leqslant x \leqslant \frac{1}{2}$.

Find, in terms of π and e, the area of the surface generated when the curve is rotated through 2π radians about the x-axis. [6]

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3 Find all the roots of the equation $(w+1)^6 = 1$, giving your answers in the form $x + \mathrm{i}y$ where x and y are real and exact. [4]

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4 Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^{x}$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. [8]

5 The curve C has equation

$$y^2 + (xy+1)^2 = 5.$$

(a) Show that, at the point $(1, 1)$ on C, $\frac{dy}{dx} = -\frac{2}{3}$. [3]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 1)$. [5]

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6 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102\cos 3t,$$

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. [11]

7 **(a)** Show that $\sum_{r=1}^{n} z^{2r} = \frac{z^{2n+1} - z}{z - z^{-1}}$, for $z \neq 0, 1, -1$. [2]

(b) By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin\theta \neq 0$,

$$1 + 2\sum_{r=1}^{n} \cos(2r\theta) = \frac{\sin(2n+1)\theta}{\sin\theta}.$$ [5]

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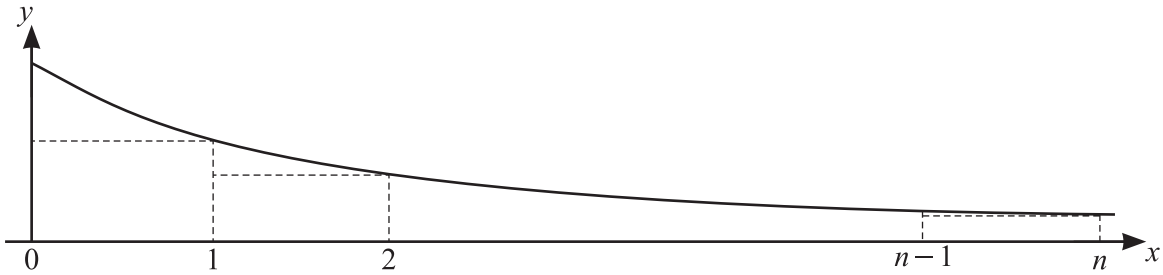

The diagram shows the curve $y = \dfrac{1}{\sqrt{x^2 + x + 1}}$ for $x \geqslant 0$, together with a set of n rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\tfrac{1}{3} + \tfrac{2}{3}n + \tfrac{2}{3}\sqrt{n^2 + n + 1}\right).$$ [10]

9 It is given that a is a positive constant.

(a) Show that the system of equations

$$\begin{aligned} ax + (2a+5)y + (a+1)z &= 1, \\ -4y \qquad &= 2, \\ 3y - z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

(b) Show that the eigenvalues of $\mathbf{A}$ are a, -1 and -4. [2]

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(c) Find a matrix $\mathbf{P}$ such that

$$\mathbf{A} = \mathbf{P}\begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}\mathbf{P}^{-1}.$$ [5]

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(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.