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9231/12

October/November 2021

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

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Let $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$.

- Given that the area of triangle DEF is 13 cm^2 , find the area of triangle PQR . [2]

- [illegible]

- [illegible]

- 2** It is given that $y = xe^{ax}$, where a is a constant.

Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n y}{dx^n} = (a^n x + n a^{n-1}) e^{ax}. \quad [6]$$

This image shows a full page of white paper with horizontal dotted lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 Let $S_n = \sum_{r=1}^n \ln \frac{r(r+2)}{(r+1)^2}$.

(a) Using the method of differences, or otherwise, show that $S_n = \ln \frac{n+2}{2(n+1)}$. [4]

This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.

$$\text{Let } S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}.$$

(b) Find the least value of n such that $S_n - S < 0.01$.

[3]

[illegible]

4 The cubic equation $x^3 + 2x^2 + 3x + 3 = 0$ has roots α , β , γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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(b) Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$. [2]

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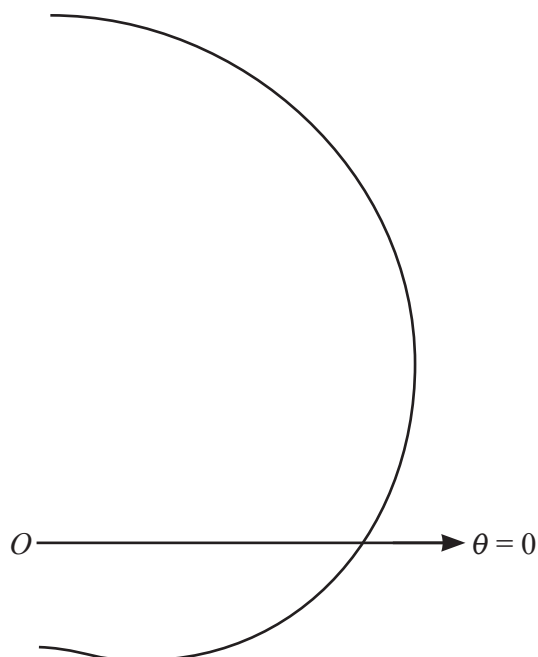
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- 5** The curve C has polar equation $r = 3 + 2 \sin \theta$, for $-\pi < \theta \leq \pi$.

- (a)** The diagram shows part of C . Sketch the rest of C on the diagram.

[1]



The straight line l has polar equation $r \sin \theta = 2$.

- (b)** Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of C and l . [5]

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- (c) The region R is enclosed by C and l , and contains the pole.

Find the area of R , giving your answer in exact form.

[6]

[illegible]

6 The curve C has equation $y = \frac{x^2}{x-3}$.

(a) Find the equations of the asymptotes of C . [3]

(b) Show that there is no point on C for which $0 < y < 12$. [4]

[illegible]

(c) Sketch C .

[2]

- (d) (i) Sketch the graphs of $y = \left| \frac{x^2}{x-3} \right|$ and $y = |x| - 3$ on a single diagram, stating the coordinates of the intersections with the axes. [4]

- (ii) Use your sketch to find the set of values of c for which $\left| \frac{x^2}{x-3} \right| \leq |x| + c$ has no solution. [1]

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- 7 The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j}, \quad -\mathbf{j} + \mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$$

respectively, relative to the origin O .

- (a) Find an equation of the plane OAB , giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

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The plane Π has equation $x - 3y - 2z = 1$.

- (b) Find the perpendicular distance of Π from the origin. [1]

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[illegible]

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