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9231/21

October/November 2022

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

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- 1 Find the Maclaurin's series for $\ln(1 + e^x)$ up to and including the term in x^2 . [5]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

- 2 (a) Show that the system of equations

$$x - y + 2z = 4,$$

$$x - y - 3z = a,$$

$$x - y + 7z = 13,$$

where a is a constant, does not have a unique solution.

[2]

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- (b) Given that $a = -5$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- (c) Given instead that $a \neq -5$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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3 The curve C has parametric equations

$$x = e^t - \frac{1}{3}t^3, \quad y = 4e^{\frac{1}{2}t}(t-2), \quad \text{for } 0 \leq t \leq 2.$$

Find, in terms of e , the length of C .

[6]

[illegible]

- 4 (a) Starting from the definitions of $\cosh$ and $\sinh$ in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1. \quad [3]$$

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- (b) Show that $\frac{d}{dx}(\tan^{-1}(\sinh x)) = \operatorname{sech} x. \quad [3]$

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- (c) Sketch the graph of $y = \operatorname{sech} x$, stating the equation of the asymptote. [2]

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- (d) By considering a suitable set of n rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^n \operatorname{sech} r < \tan^{-1}(\sinh n). \quad [3]$$

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- (e) Hence state an upper bound, in terms of π , for $\sum_{r=1}^{\infty} \operatorname{sech} r$. [1]

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5 Find the particular solution of the differential equation

$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4x^2 + 3x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

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- 7 (a) State the sum of the series $1 + w + w^2 + w^3 + \dots + w^{n-1}$, for $w \neq 1$. [1]

- (b)** Show that $(1 + i \tan \theta)^k = \sec^k \theta (\cos k\theta + i \sin k\theta)$, where θ is not an integer multiple of $\frac{1}{2}\pi$. [2]

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- (c) By considering $\sum_{k=0}^{n-1} (1 + i \tan \theta)^k$, show that

$$\sum_{k=0}^{n-1} \sec^k \theta \sin k\theta = \cot \theta (1 - \sec^n \theta \cos n\theta),$$

provided θ is not an integer multiple of $\frac{1}{2}\pi$. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (d) Hence find $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$ in terms of m . [2]

- 8 (a)** Use the substitution $u = 1 - (\theta - 1)^2$ to find

$$\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}} d\theta. \quad [3]$$

- (b)** Find the solution of the differential equation

$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = f(\theta)$. [11]

[illegible]

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