Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

8300409720

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **October/November 2022**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Find the Maclaurin's series for $\ln(1+e^x)$ up to and including the term in x^2. [5]

2 **(a)** Show that the system of equations

$$x - y + 2z = 4,$$
$$x - y - 3z = a,$$
$$x - y + 7z = 13,$$

where a is a constant, does not have a unique solution. [2]

(b) Given that $a = -5$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

(c) Given instead that $a \neq -5$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

3 The curve C has parametric equations

$$x = \mathrm{e}^t - \tfrac{1}{3}t^3, \quad y = 4\mathrm{e}^{\frac{1}{2}t}(t-2), \qquad \text{for } 0 \leqslant t \leqslant 2.$$

Find, in terms of e, the length of C. [6]

4 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1.$$ [3]

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1}(\sinh x)\right) = \operatorname{sech} x$. [3]

(c) Sketch the graph of $y = \operatorname{sech} x$, stating the equation of the asymptote. [2]

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(d) By considering a suitable set of n rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^{n} \operatorname{sech} r < \tan^{-1}(\sinh n).$$ [3]

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(e) Hence state an upper bound, in terms of π, for $\sum_{r=1}^{\infty} \operatorname{sech} r$. [1]

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5 Find the particular solution of the differential equation

$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4x^2 + 3x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$. [10]

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

(b) Use the characteristic equation of **A** to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where a and b are integers to be determined. [4]

7 **(a)** State the sum of the series $1 + w + w^2 + w^3 + \ldots + w^{n-1}$, for $w \neq 1$. [1]

(b) Show that $(1 + \mathrm{i}\tan\theta)^k = \sec^k\theta(\cos k\theta + \mathrm{i}\sin k\theta)$, where θ is not an integer multiple of $\frac{1}{2}\pi$. [2]

(c) By considering $\sum_{k=0}^{n-1}(1 + \mathrm{i}\tan\theta)^k$, show that

$$\sum_{k=0}^{n-1}\sec^k\theta\sin k\theta = \cot\theta(1 - \sec^n\theta\cos n\theta),$$

provided θ is not an integer multiple of $\frac{1}{2}\pi$. [5]

(d) Hence find $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$ in terms of m. [2]

8 **(a)** Use the substitution $u = 1-(\theta-1)^2$ to find

$$\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}}\,\mathrm{d}\theta.$$

[3]

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(b) Find the solution of the differential equation

$$\theta\frac{\mathrm{d}y}{\mathrm{d}\theta} - y = \theta^2 \sin^{-1}(\theta-1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = \mathrm{f}(\theta)$. [11]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.