



CANDIDATE
NAME

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9231/12

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1 (a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (3r^2 + 3r + 1)$ in terms of n , simplifying your answer. [3]

[illegible]

(b) Show that

$$\frac{1}{r^3} - \frac{1}{(r+1)^3} = \frac{3r^2 + 3r + 1}{r^3(r+1)^3}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{3r^2+3r+1}{r^3(r+1)^3}$. [5]

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(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{3r^2 + 3r + 1}{r^3(r+1)^3}$. [1]

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2 Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n}{dx^n}(x^2 e^x) = (x^2 + 2nx + n(n-1))e^x. \quad [6]$$

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- 3** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ and $k \neq 1$.
- (a)** The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations. State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the x - y plane is transformed by $\mathbf{M}$ onto parallelogram $OPQR$.

- (b) Find, in terms of k , the area of parallelogram $OPQR$ and the matrix which transforms $OPQR$ onto the unit square. [3]

[illegible]

- (c) Show that the line through the origin with gradient $\frac{1}{k-1}$ is invariant under the transformation in the x - y plane represented by $\mathbf{M}$. [3]

[illegible]

- 4** The cubic equation $27x^3 + 18x^2 + 6x - 1 = 0$ has roots α, β, γ .

- (a)** Show that a cubic equation with roots $3\alpha + 1$, $3\beta + 1$, $3\gamma + 1$ is

$$y^3 - y^2 + y - 2 = 0. \quad [3]$$

[illegible]

The sum $(3\alpha+1)^n + (3\beta+1)^n + (3\gamma+1)^n$ is denoted by S_n .

- (b)** Find the values of S_2 and S_3 . [4]

[illegible]

- (c)** Find the values of S_{-1} and S_{-2} . [3]

This image shows a full page of a handwriting practice worksheet. It consists of ten sets of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

5 The plane Π_1 has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

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The line l , which does not lie in Π_1 , has equation $\mathbf{r} = -3\mathbf{i} + \mathbf{k} + t(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

(b) Show that l is parallel to Π_1 . [2]

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- (c) Find the distance between l and Π_1 .

[3]

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- (d) The plane Π_2 has equation $3x + 3y + 2z = 1$.

Find a vector equation of the line of intersection of Π_1 and Π_2 .

[4]

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6 The curve C has polar equation $r = e^{-\theta} - e^{-\frac{1}{2}\pi}$, where $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

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(b) Find the exact value of the area of the region bounded by C and the initial line. [5]

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- (c) Show that, at the point on C furthest from the initial line,

$$1 - e^{\theta - \frac{1}{2}\pi} - \tan \theta = 0$$

and verify that this equation has a root between 0.56 and 0.57.

[5]

7 The curve C has equation $y = f(x)$, where $f(x) = \frac{x^2}{x+1}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of any stationary points on C . [2]

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(c) Sketch C .

[3]

(d) Find the coordinates of any stationary points on the curve with equation $y = \frac{1}{f(x)}$. [2]

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- (e) Sketch the curve with equation $y = \frac{1}{f(x)}$ and find, in exact form, the set of values for which $\frac{1}{f(x)} > f(x)$. [6]

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