



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/21

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 Show that the system of equations

$$\begin{array}{rcl} 14x - 4y + 6z & = & 5, \\ x + y + kz & = & 3, \\ -21x + 6y - 9z & = & 14, \end{array}$$

where k is a constant, does not have a unique solution and interpret this situation geometrically. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 2** Find the roots of the equation $(z + 5i)^3 = 4 + 4\sqrt{3}i$, giving your answers in the form $r \cos \theta + i(r \sin \theta - 5)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

[illegible]

- 3 Find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1}{2}e^x\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [6]

[illegible]

4 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 27x^2,$$

given that, when $x = 0$, $y = 2$ and $\frac{dy}{dx} = -8$. [10]

[illegible]

5 The curve C has parametric equations

$$x = \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}, \quad y = 2t + 5, \quad \text{for } 0 < t \leq 3.$$

(a) Find the exact length of C .

[5]

[illegible]

(b) Find the set of values of t for which $\frac{d^2y}{dx^2} > 0$. [5]

[illegible]

- 6 (a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\sinh 2x = 2 \sinh x \cosh x. \quad [3]$$

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the width of the page. There is no text or other markings on the page.

- (b)** Using the substitution $u = \sinh x$, find $\int \sinh^2 2x \cosh x \, dx$. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [4]

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- 8** (a) State the sum of the series $1 + z + z^2 + \dots + z^{n-1}$, for $z \neq 1$. [1]

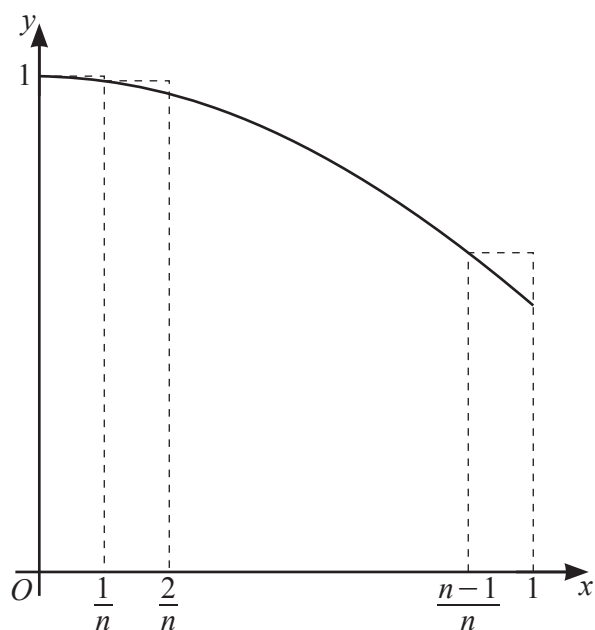
- (b)** By letting $z = \cos \theta + i \sin \theta$, where $\cos \theta \neq 1$, show that

$$1 + \cos \theta + \cos 2\theta + \dots + \cos(n-1)\theta = \frac{1}{2} \left(1 - \cos n\theta + \frac{\sin n\theta \sin \theta}{1 - \cos \theta} \right). \quad [7]$$

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The diagram shows the curve with equation $y = \cos x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(c) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 \cos x \, dx < \frac{1}{2n} \left(1 - \cos 1 + \frac{\sin 1 \sin \frac{1}{n}}{1 - \cos \frac{1}{n}} \right). \quad [4]$$

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(d) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 \cos x \, dx$. [3]

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