



CANDIDATE
NAME

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CENTRE
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CANDIDATE
NUMBER

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9231/22

October/November 2023

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 Find the Maclaurin's series for $\ln(x+2) + \ln(x^2+5)$ up to and including the term in x^2 . [5]

[illegible]

2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = te^t.$$

(a) Show that $\frac{dy}{dx} = -e^t(t^3 + t^2)$. [3]

[illegible]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t . [4]

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- 3 (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta. \quad [4]$$

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- (b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{2} = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [4]

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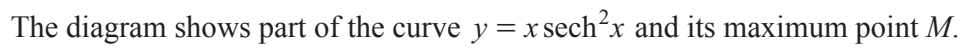
4 Find the solution of the differential equation

$$\frac{dy}{dx} + 3y = \sin x$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.

[9]

[illegible]



- and verify that this equation has a root between 0.7 and 0.8.

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(b) By considering a suitable set of rectangles, use the diagram to show that

$$\sum_{r=2}^n r \operatorname{sech}^2 r < n \tanh n + \ln \operatorname{sech} n - \tanh 1 - \ln \operatorname{sech} 1. \quad [6]$$

[illegible]

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

The 3×3 matrix $\mathbf{A}$ has distinct non-zero eigenvalues $a, \frac{1}{2}, 2$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

respectively.

- (c) Find $\mathbf{A}^{-1}$ in terms of a .

[5]

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- 7 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$2 \sinh^2 A = \cosh 2A - 1. \quad [3]$$

[illegible]

- (b)** A curve has equation $y = x^2$, for $0 \leq x \leq \frac{2}{3}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by S .

Use the substitution $x = \frac{1}{2} \sinh u$ to show that $S = \frac{1}{32} \pi \left(\frac{820}{81} - \ln 3 \right)$. [9]

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8 It is given that $v = y^4$ and

$$y^3 \frac{d^2 y}{dx^2} + 3y^2 \left(\frac{dy}{dx} \right)^2 + y^3 \frac{dy}{dx} + y^4 = e^{-2x}.$$

(a) Show that

$$\frac{d^2v}{dx^2} + \frac{dv}{dx} + 4v = 4e^{-2x}. \quad [4]$$

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(b) Find y in terms of x , given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -\frac{3}{8}$. [10]

[illegible]

[illegible]

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