



# Cambridge International AS & A Level

CANDIDATE  
NAME



CENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

## FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.



Prove by induction that  $u_n = 3^n + 1$  for all positive integers  $n$ . [5]

[5]

[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



5

- [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

[illegible]

$$\begin{aligned}\alpha + \beta + \gamma + \delta &= 2, \\ \alpha^2 + \beta^2 + \gamma^2 + \delta^2 &= 3, \\ \alpha^3 + \beta^3 + \gamma^3 + \delta^3 &= 4.\end{aligned}$$

(a) Find the value of  $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ . [2]

**(b)** Find the value of  $\alpha^2\beta + \alpha^2\gamma + \alpha^2\delta + \beta^2\alpha + \beta^2\gamma + \beta^2\delta + \gamma^2\alpha + \gamma^2\beta + \gamma^2\delta + \delta^2\alpha + \delta^2\beta + \delta^2\gamma$ . [3]



(c) It is given that  $\alpha, \beta, \gamma, \delta$  are the roots of the equation

$$6x^4 - 12x^3 + 3x^2 + 2x + 6 = 0.$$

- (i) Find the value of  $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Find the value of  $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

(a) Show that  $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$ . [3]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the  $x$ - $y$  plane represented by **CAB**. [5]



5

- 
- 
- 
- 
- 

- [illegible]



5 It is given that  $S_n = \sum_{r=1}^n u_r$ , where  $u_r = x^{f(r)} - x^{f(r+1)}$  and  $x > 0$ .

(a) Find  $S_n$  in terms of  $n$ ,  $x$  and the function  $f$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that  $f(r) = \ln r$ , find the set of values of  $x$  for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



[illegible]

[illegible]



- (d) Sketch  $C$ , stating the coordinates of any intersections with the axes and labelling the asymptote. [3]

.....

- (e) Sketch the curve with equation  $y = \frac{x^2 + 1}{x^2 + 3}$  and find the set of values of  $x$  for which  $\frac{x^2 + 1}{x^2 + 3} < \frac{1}{2}$ . [4]

.....  
 .....  
 .....  
 .....





7 The curve  $C_1$  has polar equation  $r = a(\cos \theta + \sin \theta)$  for  $-\frac{1}{4}\pi \leq \theta \leq \frac{3}{4}\pi$ , where  $a$  is a positive constant.

(a) Find a Cartesian equation for  $C_1$  and show that it represents a circle, stating its radius and the Cartesian coordinates of its centre. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Sketch  $C_1$  and state the greatest distance of a point on  $C_1$  from the pole. [3]

.....

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





5

- [illegible]

- DO NOT WRITE IN THIS MARGIN

and deduce, in terms of  $a$  and  $\phi$ , the area of the larger region enclosed by  $C_1$  and  $C_2$ . [7]

DO NOT WRITE IN THIS MARGIN

[illegible]



This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.





**BLANK PAGE**

DO NOT WRITE IN THIS MARGIN





**BLANK PAGE**

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

