



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





- 1 Ellie is investigating the heights of two types of beech tree, A and B , in a certain region. She has chosen a random sample of 60 beech trees of type A in the region, recorded their heights, x m, and calculated unbiased estimates for the population mean and population variance as 35.6 m and 4.95 m^2 respectively.

Ellie also chooses a random sample of 50 beech trees of type *B* in the region and records their heights, ym. Her results are summarised as follows.

$$\Sigma y = 1654 \quad \Sigma y^2 = 54\,850$$

Find a 95% confidence interval for the difference between the population mean heights of type A and type B beech trees in the region. [6]

[illegible]

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

[illegible]

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

[illegible]

[illegible]



4 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx^3 & 0 \leq x < 1, \\ k(5-x) & 1 \leq x \leq 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Sketch the graph of f .

[1]

(b) Show that $k = \frac{4}{33}$.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





(c) Find the cumulative distribution function of X .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Find the median value of X .

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





- 5 Nikita has three coins. One coin is fair, one coin is biased so that the probability of obtaining a head is $\frac{1}{3}$ and the third coin is biased so that the probability of obtaining a head is $\frac{1}{5}$. The random variable X is the number of heads that Nikita obtains when he throws all three coins at the same time.

(a) Find the probability generating function of X . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Rajesh has two fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6. The random variable Y is the number of 4s that Rajesh obtains when he throws the two dice.

The random variable Z is the sum of the number of heads obtained by Nikita and the number of 4s obtained by Rajesh.

(b) Find the probability generating function of Z , expressing your answer as a polynomial. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





DO NOT WRITE IN THIS MARGIN

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Use your answer to part (b) to find $E(Z)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 8.2 7.0 7.3 8.8 7.8 8.5 9.2 7.4

$$\Sigma y = 71.10 \quad \Sigma y^2 = 567.13$$

Test, at the 10% significance level, whether Ansal's suspicion is supported by the data. [8]

[illegible]



Handwriting practice area with 20 horizontal dotted lines.



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]



DO NOT WRITE IN THIS MARGIN





BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

