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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (r^3 - r)$ in terms of n , fully factorising your answer. [3]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing letters. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



- (b) Express $\frac{r+3}{r^3-r}$ in the form $\frac{A}{r-1} + \frac{B}{r} + \frac{C}{r+1}$, where A , B and C are constants to be determined, and hence use the method of differences to find $\sum_{r=2}^n \frac{r+3}{r^3-r}$. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) Deduce the value of $\sum_{r=2}^{\infty} \frac{r+3}{r^3-r}$. [1]





- 2** The cubic equation $x^3 + bx^2 + cx + d = 0$, where b , c and d are constants, has roots α , β and γ . It is given that

$$\begin{aligned}\alpha + \beta + \gamma &= 2, \\ \alpha^2 + \beta^2 + \gamma^2 &= 3, \\ \alpha^4 + \beta^4 + \gamma^4 &= 5.\end{aligned}$$

- (a) Find the values of b and c .

[3]

[illegible]



(b) Find the value of d .

[5]

[illegible]



- 3 The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 < 5$ and, for $n \geq 1$,

$$u_{n+1} = \frac{6u_n + 5}{u_n + 2}.$$

- (a) By considering $5 - u_{n+1}$, prove by mathematical induction that $u_n < 5$ for all positive integers n . [5]

[illegible]



(b) Show that $u_{n+1} > u_n$ for $n \geq 1$.

[3]





4 Let k and m be non-zero constants. The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the x - y plane represented by the matrix $\mathbf{B}$ in each of the following cases.

- (i) $m = 1$ [2]

- (ii) $m = k$ [2]

- (b) Show that the matrix $\mathbf{ABC}$ is singular. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.





5 The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leq \theta \leq \frac{1}{8}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole.

[2]

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(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$.

[4]

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- (c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leq x \leq \cos\left(\frac{1}{8}\pi\right)$ and $0 \leq y \leq \sin\left(\frac{1}{8}\pi\right)$.

[4]

- (d) Using your answer to (b), deduce the exact value of the area bounded by C , the x -axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

[2]





6 The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π . [2]

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Relative to O , the points A, B, C have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π . [4]

[illegible]



(c) Find an equation for the common perpendicular to the lines OC and AB .

[8]

[illegible]



7 The curve C has equation $y = \frac{x^2 + x + 1}{x + 1}$.

(a) Find the equations of the asymptotes of C .

[3]

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(b) Find the coordinates of any stationary points on C .

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(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \frac{|x|^2 + |x| + 1}{|x| + 1}$.

[2]





(e) Find, in exact form, the set of values of x for which $\frac{|x|^2 + |x| + 1}{|x| + 1} < 3$. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]







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