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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER						
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FURTHER MATHEMATICS **9231/22**

Paper 2 Further Pure Mathematics 2 **October/November 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1 The curve C has polar equation $r = e^{\frac{4}{3}\theta}$ for $0 \leqslant \theta \leqslant \pi$.

Find the length of C. Give your answer in an exact form. [4]

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2 Find the roots of the equation $z^4 = 8 - 8\mathrm{i}\sqrt{3}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

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3 The variables x and y are such that $y = 2$ when $x = -1$ and

$$x^2y + (x+y)^3 = 3.$$

(a) Show that $\frac{dy}{dx} = \frac{1}{4}$ when $x = -1$. [3]

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(b) Find the value of $\frac{d^2y}{dx^2}$ when $x = -1$. [6]

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4 **(a)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)^6$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^6\theta = a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d,$$

where a, b, c and d are constants to be determined. [5]

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(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 2x \, dx$. [3]

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5 **(a)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=5\cos x.$$ [7]

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(b) For large positive values of x and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R\sin(x+\phi),$$

where the constants R and ϕ are to be determined. [3]

6 **(a)** Use the substitution $x = \frac{1}{2}\sqrt{2}\sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}}\,dx$. [3]

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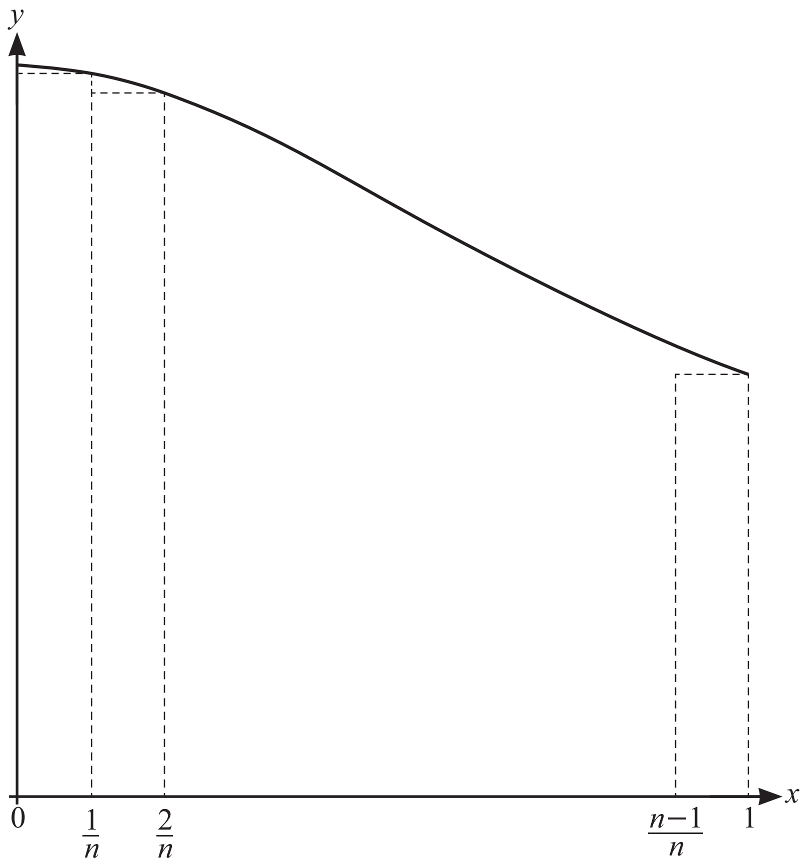

The diagram shows the curve with equation $y = \frac{1}{\sqrt{2x^2+1}}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2}\ln\left(\sqrt{2}+\sqrt{3}\right).$$ [5]

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(c) Use a similar method to find, in terms of n, a lower bound for $\sum_{r=1}^{n} \frac{1}{\sqrt{2r^2 + n^2}}$. [4]

(d) Deduce the exact value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{\sqrt{2r^2 + n^2}}$. [1]

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7 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}x\sqrt{4-x^2}+2\sin^{-1}\left(\frac{1}{2}x\right)\right)=\sqrt{4-x^2}$. [3]

(b) Find the solution of the differential equation

$$2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{y}{2+x}=2\sqrt{2-x}$$

for which $y=\frac{1}{2}$ when $x=1$. Give your answer in an exact form. [8]

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8 **(a)** Find the values of k for which the system of equations

$$x - y + 2kz = 1,$$
$$kx + y + 2z = 2,$$
$$2x - y + z = 3,$$

does not have a unique solution. [3]

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(b) Given that $k = -\frac{1}{2}$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [3]

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(c) Given instead that $k = -1$, show that the system of equations in part **(a)** is also inconsistent. Interpret this situation geometrically. [4]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}.$$

(d) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A}$ where p and q are integers to be determined. [5]

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Additional page

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