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FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B . After the collision, the momentum of A is three times the momentum of B .

Find the value of the coefficient of restitution e .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r . The angle between the upward vertical and the normal reaction of the surface on P is θ_1 , where $\tan \theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2 , the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2 , where $\tan \theta_2 = \frac{4}{3}$.

Find the ratio $\frac{\omega_1}{\omega_2}$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



- 3** A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is v ms⁻¹. The acceleration of P is $\frac{1}{2}(v^2 + 4)$ ms⁻² in the direction PO . Initially P is at O and is moving with velocity 2 ms⁻¹.

(a) Find an expression for x in terms of t . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Find the time when P next goes through O .

[2]





- 4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O . Two particles, P and Q , of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

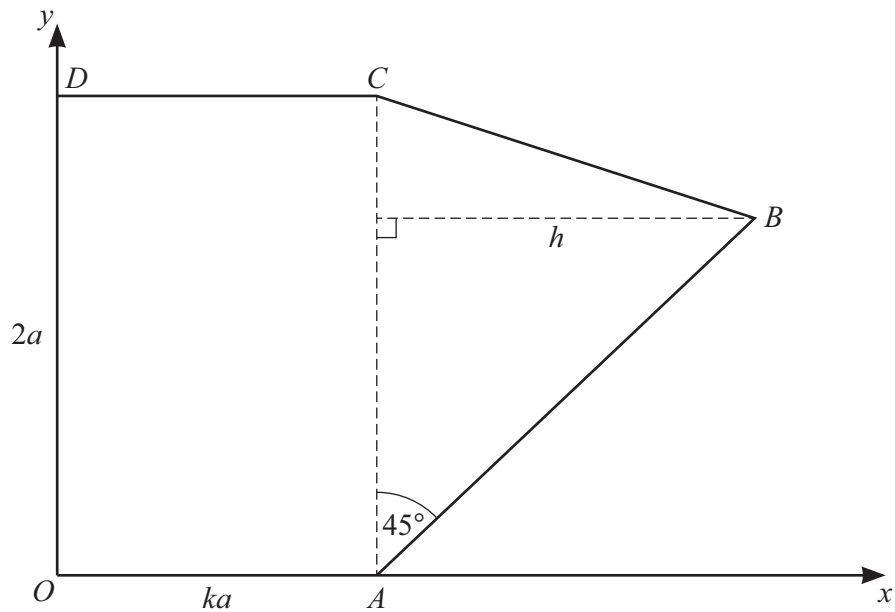
Find, in terms of a , the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



7

[illegible]



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC . The length of OA is ka , the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O , parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\bar{x}, \bar{y})$.

- (a) Show that $\bar{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\bar{y}$. [3]

[illegible]



The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

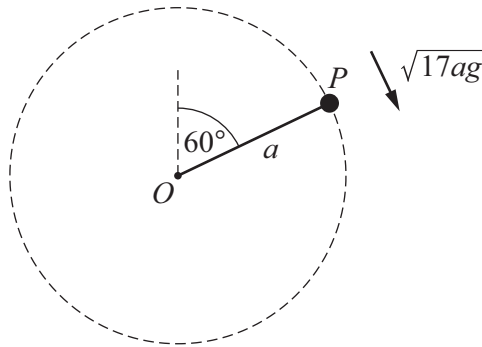
- (b) Find, in terms of a and k , the set of values of h for which the lamina is in equilibrium. [4]

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It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

- (c) Find, in terms of a , the coordinates of the centre of mass of the triangle ABC . [2]

[illegible]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . Initially P is held with the string taut and making an angle of 60° with the upward vertical through O . The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q .

- (a) Find, in terms of a and g , an expression for the speed of P immediately before the collision with Q . [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



As a result of the collision, P rebounds and moves back along a circular path with centre O . The string becomes slack when P reaches the point on the circle vertically above O .

- (b) Find, in terms of a and g , an expression for the speed of P immediately after the collision with Q . [3]

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The mass of particle Q is km and the collision between P and Q is perfectly elastic.

- (c) Find the value of k . [3]

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- 7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 25ms^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A , the direction of motion of P makes an angle of 45° with the downward vertical through A .

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



At point A , the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

- (b) Find the speed of P immediately before it collides with the barrier. [3]

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- (c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

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[illegible]





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