Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER



**FURTHER MATHEMATICS** **9231/34**

Paper 3 Further Mechanics **October/November 2025**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.




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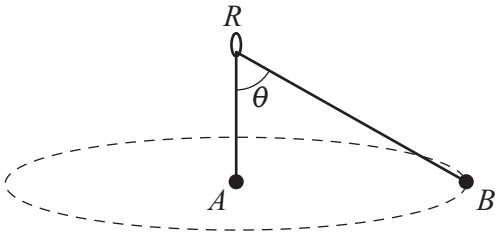
1




A light inextensible string of length $12a$ is threaded through a fixed smooth ring $R$. One end of the string is attached to a particle $A$ of mass $m$. The other end of the string is attached to a particle $B$ of mass $0.5m$. Particle $A$ hangs in equilibrium vertically below the ring. Particle $B$ moves with constant angular speed $\omega$ in a horizontal circle with particle $A$ at its centre. The angle between $AR$ and $BR$ is $\theta$ (see diagram).

Express $\omega$ in terms of $g$ and $a$. [5]

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**2** A particle $P$ is projected with speed $u\,\text{m s}^{-1}$ at an angle $\theta$, where $\tan\theta = 2$, above the horizontal from a point $O$ on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of $P$ from $O$ at a time $t$ s are denoted by $x$ m and $y$ m respectively.

**(a)** Use the equation of the trajectory given in the list of formulae (MF 19) to show that

$$y = 2x - \frac{25x^2}{u^2}\ .$$

[1]

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**(b)** In the subsequent motion, $P$ passes through the point with coordinates (8,12). The particle then hits a fixed vertical barrier 7 m high that is at a horizontal distance of $D$ m from the point of projection.

Find the set of possible values of $D$. [5]

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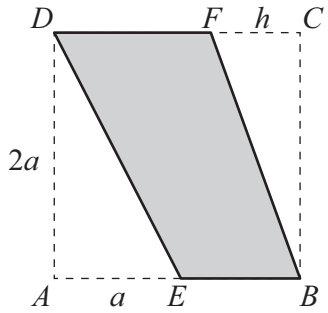


The lamina $BFDE$ is obtained by removing triangles $AED$ and $BCF$ from a uniform square lamina $ABCD$ of side $2a$. The length of side $AE$ is $a$ and the length of side $FC$ is $h$ (see diagram). The centre of mass of $BFDE$ is at a distance $\overline{x}$ from $AD$, and at a distance $\overline{y}$ from $AB$.

**(a)** Show that $\overline{x} = \dfrac{h^2 - 6ah + 11a^2}{3(3a - h)}$ and find a corresponding expression for $\overline{y}$. [5]

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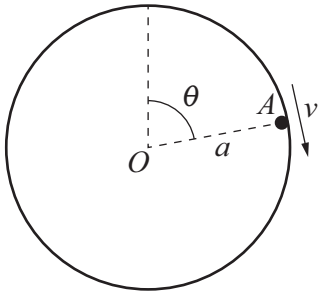
**(b)** The lamina *BFDE* is placed vertically on its edge *EB* on a smooth horizontal surface.

Find, in terms of $a$, the set of possible values of $h$ for which the lamina remains in equilibrium. [2]

**4**




A fixed smooth spherical shell has centre $O$ and radius $a$. A particle of mass $m$ moves in complete vertical circles on the smooth inner surface of the shell, where the plane of the circular motion is vertical and passes through $O$. The particle has speed $v$ when it is at point $A$, where $OA$ makes an angle $\theta$ with the upward vertical through $O$, and $\cos\theta = \frac{1}{18}$ (see diagram).

**(a)** Show that $v \geqslant \frac{1}{3}\sqrt{26ag}$. [5]

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It is given that $v = \frac{1}{3}\sqrt{26ag}$.

**(b)** Find, in terms of $m$ and $g$, an expression for the greatest possible value of the normal reaction between the shell and the particle. [3]

**5** A particle $P$ of mass $m$ kg is projected vertically upwards from a point $O$ with an initial speed of $20\,\text{m s}^{-1}$ and moves under gravity. There is a resistive force of magnitude $0.025mv^2$ N, where $v\,\text{m s}^{-1}$ is the speed of $P$ at time $t$ s after projection. The displacement of $P$ from $O$ is $x$ m at time $t$ s after projection.

**(a)** Find an expression for $v$ in terms of $x$, while $P$ is moving upwards. [5]

**(b)** Find an expression for $v$ in terms of $t$, while $P$ is moving upwards. [4]

**6** $A$ and $B$ are two fixed points at a distance $22a$ apart, with $B$ vertically below $A$. A light elastic string of natural length $4a$ and modulus of elasticity $5mg$ has one end attached to $A$ and the other end attached to a particle $P$ of mass $km$. Another light elastic string of natural length $8a$ and modulus of elasticity $6mg$ has one end attached to $B$ and the other end attached to $P$. Particle $P$ is vertically above $B$.

**(a)** Show that, when the system is in equilibrium, $BP = \frac{57a - 2ak}{4}$. [5]

The particle $P$ is pulled vertically upwards so that $BP = 18a$, and is then released from rest. In its subsequent motion, $P$ first comes to instantaneous rest at the point where $BP = 8a$.

**(b)** Find the value of $k$. [4]

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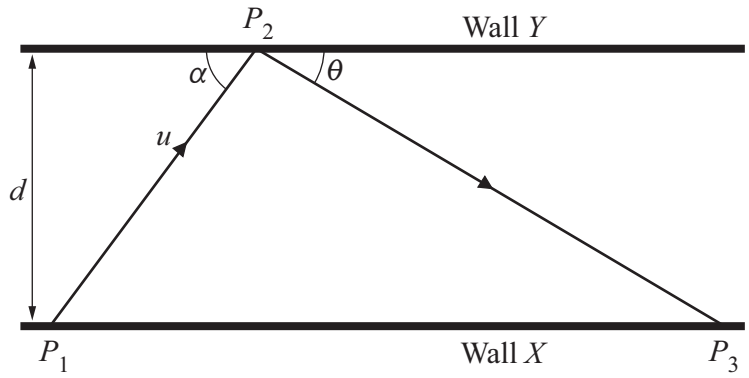


$X$ and $Y$ are two fixed smooth vertical walls on a smooth horizontal surface. The walls are parallel and at a distance $d$ apart. The points $P_1$, $P_2$ and $P_3$ all lie on the surface.

A particle $Q$ is projected horizontally from the point $P_1$ on Wall $X$ with speed $u$, and moves along the surface. The particle $Q$ strikes Wall $Y$ at the point $P_2$. Immediately before the collision, the direction of motion of $Q$ makes an angle $\alpha$ with Wall $Y$, where $\sin\alpha = \frac{4}{5}$. Immediately after the collision, the direction of motion of $Q$ makes an angle $\theta$ with Wall $Y$. The particle $Q$ then strikes Wall $X$ at the point $P_3$ (see diagram).

The time that it takes $Q$ to travel the distance $P_1P_2$ is $T$. The time that it takes $Q$ to travel the distance $P_2P_3$ is $kT$.

Find, in terms of $k$, the coefficient of restitution between $Q$ and Wall $Y$. [6]

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**Additional page**

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