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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



- 1 A group of 10 school children are asked to estimate the size of an angle θ° in a given acute angled triangle. These estimates, in degrees, are as follows.

84 85 77 85 84 87 86 88 83 85

- (a) Stating any assumptions you make, calculate a 95% confidence interval for θ . [5]

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- (b)** Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

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- 2 The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2 -test statistic.

Number of cars	0	1	2	3	4	5	≥ 6
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2 -test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

- (a) Find the value of p and the value of q . [2]

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- (b)** Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

[illegible]



- 3.9 3.1 2.9 3.1 3.6

By pooling the two samples, calculate an estimate of σ .

[4]

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- 4 A researcher believes that the median m of a population has changed from its known previous value m_0 . The researcher collects a random sample of size 28. She ranks the data and calculates a test statistic T using the Wilcoxon signed-rank test. The conclusion of the test carried out at a 1% significance level is that there is not sufficient evidence to support her belief.

Using a normal approximation, find the least possible value of T . [5]

[illegible]



5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leq x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leq x \leq 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$.

[2]

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(b) Find the median value of X .

[3]

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The random variable Y is defined by $Y = \sqrt{X}$.

- (c) Find the probability density function of Y .

[5]

[illegible]



- 6 Nine athletes in a club have a new coach. The coach adopts a new training programme which he believes will reduce the race times of these athletes. Each athlete completes a 1500m time trial before and after completing the new training programme. Their times, in seconds (s), are recorded.

Athlete	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
Time before training (s)	250	251	252	267	276	291	310	320	335
Time after training (s)	245	251	253	261	275	293	302	313	320

- (a) Carry out a paired t -test at the 5% significance level to test the coach's belief. [7]

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Further research suggests that the effects of the training programme tend to reduce the times of the slower athletes by more than those of the faster athletes.

- (b) Suggest a reason why the paired t -test used in part (a) may not have been an appropriate test in this case. [1]

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- (c) Suggest a suitable alternative test that could have been used instead of a paired t -test. [1]

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- 7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $P(X = r)$ as given in the following table.

r	0	1	2
$P(X=r)$	a	$2a$	b

- (a) Write down the probability generating function of X , and use it to find an expression for $E(X)$ in terms of a and b . [2]

[illegible]

- (b)** Show that $\text{Var}(X) = 2b + 2(a+b)(1-2a-2b)$. [3]

[illegible]



The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \dots + X_{10}$ where $X_1, X_2, X_3, \dots, X_{10}$ are ten independent observations of X .

- (c) Using the probability generating function of Y , and your answer to part (a), show that $E(Y) = 10E(X)$. [3]

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- (d) For the case $b = 0$, define fully the distribution of Y . [2]

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Additional page

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