



Cambridge International AS & A Level

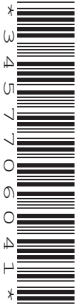
THINKING SKILLS

9694/33

Paper 3 Problem Analysis and Solution

May/June 2022

2 hours



You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

- 1 There is a train service between Arba and Boab. Details of the different types of train ticket available are shown in the table below.

<i>Type of ticket</i>	<i>Restrictions on use</i>	<i>Cost</i>
Single	One journey in either direction (Arba to Boab OR Boab to Arba)	\$4.50
Day return	One journey in each direction on the same day (Arba to Boab AND Boab to Arba)	\$7.50
5* weekly	5 single journeys in either direction in the same week	\$20.00
Weekly return	5 journeys in each direction in the same week	\$36.00
Weekend special	Any two journeys, in either direction, both on Saturday, both on Sunday or one on Saturday and one on Sunday	\$5.00

Jacob and Katy live in Arba and travel by train to and from work in Boab. Each of them makes all of the journeys allowed by each ticket that he or she buys, and does not make any other journeys by train.

Jacob works on Mondays, Tuesdays and Wednesdays.

- (a) What are the five possible costs that Jacob could pay for train journeys in one week? [2]

Katy works on Tuesdays, Wednesdays, Thursdays, Fridays and Saturdays.

- (b) What is the least possible cost that Katy could pay for train journeys in one week? State how she would achieve this. [2]

Trains are not very reliable, and often arrive late on weekdays. However, they are never late on Saturdays and Sundays.

There is a compensation scheme when trains arrive more than 15 minutes late. Currently, any customer on a train that arrives late at the customer's destination can claim \$1 for that journey. However, a new system has been proposed: instead of the separate \$1 claims, customers whose trains arrived late on 10 or more occasions in any year can now claim a voucher giving one week of free travel in the following year. Only one such claim may be made each year.

Katy works 40 weeks in a year. She wants to work out the impact of the change to the compensation scheme on her travel costs. She assumes that between 10% and 20% of her trains will arrive late.

- (c) Based on her assumption:

- (i) What is the greatest amount that Katy could claim in compensation in one year under the current scheme? [1]

- (ii) Show that Katy's travel costs could be lower by at most \$3 under the proposed scheme. [2]

The charges for train journeys are simplified as shown below.

<i>Type of ticket</i>	<i>Restrictions on use</i>	<i>Cost</i>
Single	One journey in either direction	\$5
Day return	One journey in each direction on the same day	\$9
Weekly	Any number of journeys in either direction in a period of seven days	\$50

Donald also lives in Arba. He gets a job in Boab for four weeks in April. Each week starts on a Monday and he must work six days in each week, but he can choose which six days they are. This year, 1 April is a Monday.

(d) (i) What is the least possible total cost of Donald's journeys to and from work in April? [2]

(ii) Donald achieves this least possible cost, and chooses not to work on Monday 1 April.

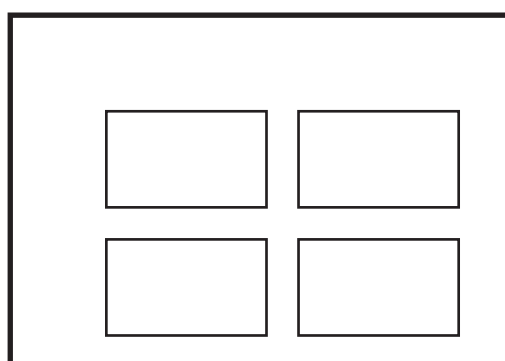
What is the latest possible date of the next day on which he will not work? [1]

- 2 Penelope is organising an exhibition to show the paintings of her class of art students. Each painting measures $30\text{ cm} \times 50\text{ cm}$.

Penelope will buy display boards that are 2.0 m tall to display her students' paintings. She has not yet decided what width(s) to buy. The boards will be placed around the edge of the room, so only one side of each board can be used to display paintings.

Penelope plans to display the paintings with the longer side horizontal. She will arrange the paintings in vertical columns with the edges aligned.

There must be a gap of at least 30 cm between any painting and the edge of the display board on which it is placed. There must be a gap of at least 10 cm between any two paintings.



- (a) (i) Show that the largest number of paintings that could be displayed in one column on a display board is 3. [2]
- (ii) If the gaps between paintings in one column of 3 paintings were all the same height, what is the maximum this height could be? [1]

Penelope has 20 students. Each student will produce 2 paintings for the exhibition.

- (b) If Penelope were to buy display boards with a width of 3.0 m , how many display boards would be needed to show all the paintings? [2]

The widths of boards that are available and their prices are shown in the table. (There are plenty of each width in stock.)

<i>Width (m)</i>	2.0	2.5	3.0	3.5	4.0
<i>Price (\$)</i>	50	55	60	65	70

- (c) What is the least that Penelope could pay to buy enough display boards to show all of the paintings? [3]

Unfortunately, the students did not listen to Penelope's instructions and each student has painted one picture that needs to be displayed with the longer side horizontal and one picture that needs to be displayed with the shorter side horizontal. Penelope would like to place the pictures so that, for any display board, all the pictures are oriented the same way.

- (d) (i) What is the least that Penelope could pay to buy enough display boards to display all 20 paintings that will have the longer side horizontal? [1]
- (ii) What is the least that Penelope could pay to buy enough display boards to display all 20 paintings that will have the shorter side horizontal? [2]

Penelope decides instead that paintings can be placed in both orientations on each display board, but they will be arranged in columns within which all paintings are in the same orientation. There must still be a gap of at least 10 cm between each column of paintings.

- (e) What is the minimum that Penelope could pay to buy enough boards to display all of the paintings? For each board, state how many columns of each orientation there would be. [4]

- 3 Moses is researching his family history and is particularly interested in five siblings (brothers and sisters) who lived a century ago. He finds five letters, each of which was written by one of the five siblings to one of the other four. Unfortunately, the letters are addressed to their nicknames, and it is not clear which nickname refers to which sibling. Moses is sure that each of the five nicknames refers to a different one of the five siblings and wants to match the siblings to their nicknames.

Below are names of the five siblings, their gender (M/F) and the nicknames they addressed letters to:

Alfred (M)	wrote a letter to	Pozzle
Bede (M)	wrote a letter to	Quiggie
Celeste (F)	wrote a letter to	Rusty
Dorian (M)	wrote a letter to	Soppet
Ethel (F)	wrote a letter to	Tupper

Initially Moses suspects that the two sisters wrote to each other.

- (a) Assuming that Moses is correct, list the two possible ways in which the three brothers could be matched to their nicknames. [1]

Moses knows that he can match any nickname to the correct sibling if he studies the letter addressed to that person carefully. He studies the letter written by Celeste and finds out that his initial suspicion was wrong: Rusty is in fact Alfred.

Moses begins to construct a table showing the ways in which the other four nicknames could be matched to the siblings. He uses the first letter of each name to save space.

<i>Pozzle</i>	B	B	B	...
<i>Quiggie</i>	C	D	E	...
<i>Rusty</i>	A	A	A	...
<i>Soppet</i>	E	E	C	...
<i>Tupper</i>	D	C	D	...

- (b) Copy Moses' table and complete it with all the other possible ways to match the nicknames to the siblings. [3]

Moses wants to study the smallest number of letters possible, as each one takes a long time to study carefully. He considers what might happen if he now studies the letter written by Alfred, to determine the identity of Pozzle.

- (c) (i) Suppose that Moses finds that Pozzle is Bede. With reference to Moses' table, explain which of the letters is **the one** he should study next to be able to match all the nicknames to the correct siblings. [2]
- (ii) Suppose that Moses finds that Pozzle is Celeste. With reference to Moses' table, explain why he will be able to match all the nicknames to the correct siblings by studying **any one** of the other letters. [1]

Moses decides that, instead of studying the letter written by Alfred, he will read all of the letters quickly to see if he can find any clues. After doing this, he concludes that:

- no two of the letters form a pair in which two siblings wrote to each other (for example, if Alfred wrote to Bede, then Bede did not write to Alfred);
- Pozzle is one of the sisters.

Moses decides to study the letter written by Dorian, and finds that Soppet is Bede.

- (d) Deduce how all the remaining nicknames are matched to the siblings. Explain each step in your reasoning. [3]

- 4 George has his own window cleaning business. His charges to customers depend on the type of building and the number of windows. These charges, and the time taken to complete the job, are shown in the following table.

<i>Type of building</i>	<i>Number of windows included in basic charge</i>	<i>Basic charge</i>	<i>Time taken (minutes)</i>
House	12	\$36	40
Bungalow	6	\$20	25
Apartment	5	\$12	20

Extra windows are charged at \$3 each and take 4 minutes each.

George has a contract to clean all the windows on the Riverside estate, once a month. There are 30 houses, each with 15 windows, and 10 bungalows, each with 6 windows.

- (a) (i) Show that the total income that George will take from Riverside each month is \$1550. [1]
- (ii) Find the total time taken, in minutes, to clean the windows in Riverside each month. [1]

George also cleans all the windows on the Lakeview estate. There are 50 houses, 30 bungalows and 15 apartments. All the windows in these buildings are included in the basic charge.

George works at least $6\frac{1}{2}$ hours a day and no more than 7 hours a day, excluding breaks and travel time between buildings. He will only start on a building if he has time to finish it that day.

- (b) (i) Find the greatest possible income on the first day of cleaning windows on Lakeview. [2]
- (ii) Find the least possible income on the first day of cleaning windows on Lakeview. [3]

The Waterfall estate consists of 240 houses and 120 apartments, all with windows that come within the basic charge. George decides to recruit sufficient employees so that all the windows of the buildings on Waterfall can be cleaned within a working week of 7 hours a day for 5 days. He will not clean any of these windows himself.

- (c) How many employees does George need to recruit? Justify your answer. [3]

Business is so good that George decides to increase his number of employees to 10. He will not clean windows himself. He will simplify his charges to \$30 for any building and allow 40 minutes per building. Each employee will bring in an income of \$1500 per week and be paid \$1000 per week. The other costs (materials, insurance, etc.) amount to \$250 per week per employee.

- (d) (i) How much time will each employee spend working every week? [1]
- (ii) Find George's weekly profit. [1]

George decides to invest in a new method of cleaning windows, the 'water-fed pole', which means that all windows can be cleaned from ground level. He estimates that this will result in a 25% reduction in the time taken to clean each building. He will also increase his charges by 10%.

George will pay each employee \$1000 per week, for all 52 weeks of the year. Each employee will work for 45 weeks in the year, the remaining time being holiday. The other costs will be \$500 per week per employee, whether or not the employee is working or on holiday. When working, each employee will clean windows for 6 hours each day, 5 days a week.

- (e) George wants his profit to be at least \$80 000 per year. Find the smallest number of employees that he will need to employ. Justify your answer. [3]

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