



Cambridge International AS & A Level

THINKING SKILLS

9694/31

Paper 3 Problem Analysis and Solution

May/June 2023

2 hours



You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.

Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.

In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **8** pages. Any blank pages are indicated.

- 1 *OrienT-8* is a single-player game, played on a 10×10 grid displayed on the touch screen of an electronic device.

The game consists of five rounds. In each round the grid contains eight T-shapes, each occupying four squares.

- In round one, four are revealed at the start and the player has to find the other four.
- In round two, three are revealed at the start and the player has to find the other five.
- In round three, two are revealed at the start and the player has to find the other six.
- In round four, one is revealed at the start and the player has to find the other seven.
- In round five, none are revealed at the start and the player has to find all eight.

At no time do two T-shapes ever touch, either edge to edge or corner to corner.

In every round:

- When a square is touched, either a tick (✓) appears in the square, indicating that part of a T-shape occupies the square, or a cross (X) appears. Each tick scores 2 points, whereas each cross deducts 1 point from the player's score.
- Immediately after a tick appears that completes a T-shape, all four squares turn black and a bonus of 2 points is added to the score.
- The round ends when all eight T-shapes have been revealed or when twelve crosses have appeared, whichever occurs first.

No points are scored for T-shapes already displayed at the start of any round.

Tom is playing a game of *OrienT-8*. This is the current situation part way through round three.

91	92	93	94	95	96	97	98	X	
81	82		84	85	86	X	X		
71				75	76	✓	78	79	
61	X	63	64	65	66	✓	68	69	70
X	52	53	54	55	56	57	58	59	60
41	42	43	44	45	X	47			
	32	33	34	35		37	38		40
		23	24				28	29	30
	12	13	14	15	X	17	18	19	20
1	2	3	4	5	6	7	8	9	10

The squares have been numbered to help identify positions on the grid. For instance the T-shape in the top right corner can be described as 80-89-90-100.

Tom has completed both of the first two rounds without any crosses appearing at all. He knows that he can complete this round without any further crosses and he is hopeful that he can beat his previous best total score of 273.

- (a) How many squares has Tom touched so far this round? [2]
- (b) What is Tom's total score at present? [2]
- (c) What evidence is there that 11-21-22-31 and 39-48-49-50 are the two T-shapes that were revealed at the start of round three? [1]
- (d) Give the numbers of the ten squares that Tom will touch to complete the last three T-shapes in this round. [3]
- (e) In order to register a new personal best score, what is the maximum number of crosses that can be revealed altogether in the last two rounds? [2]

- 2 A 'three-legged race' is a running event in which pairs of participants compete with the left leg of one of each pair strapped to the right leg of the other. Today is Bryford's annual carnival. The highlight of the carnival every year is a series of five three-legged races in which three teams of six compete for the Tripod Trophy. The teams are Team Blue, Team Red and Team Yellow.

In each race, all 18 participants take part, paired with another member of their own team. Points are awarded to the first five pairs to cross the finishing line, as follows:

First	12 points
Second	8 points
Third	5 points
Fourth	3 points
Fifth	1 point

On the rare occasions that two or more pairs cross the finishing line together, the pairs involved run again to decide the positions, but only if at least one of the teams involved will score any points as a result.

No-one is allowed to be paired with the same person twice, so every participant competes once with every other member of their team.

In addition to the trophy awarded to the winning team, the individual participant with the greatest number of points wins a cash prize. The points awarded to a pair count only once towards the team trophy in each race, but both partners are awarded the points towards their individual totals.

This is today's scoreboard, showing the points awarded to the participants in the first three races.

<i>Surname</i>	<i>Team</i>	<i>Race 1</i>	<i>Race 2</i>	<i>Race 3</i>	<i>Race 4</i>	<i>Race 5</i>	<i>Total</i>
Amber	Yellow	3		8			
Brick	Red		5	5			
Cherry	Red	5					
Denim	Blue		8				
Flame	Red	12		5			
Honey	Yellow			12			
Lemon	Yellow	8					
Madder	Red	12	1				
Mustard	Yellow	3		12			
Ocean	Blue	1	8	3			
Ochre	Yellow						
Peacock	Blue		12	3			
Royal	Blue		3				
Ruby	Red		1	1			
Saffron	Yellow	8		8			
Scarlet	Red	5	5	1			
Slate	Blue		12				
Teal	Blue	1	3				

Team Blue is the only one of the three teams that has never won the Tripod Trophy, and they made a poor start today, scoring only 1 point in the first race.

- (a) (i) How many points did Team Red score and how many points did Team Yellow score in the first race? [2]

- (ii) Who was Honey's partner in the first race? [1]

The results of the fourth race, which has just finished, are as follows:

First	Flame & Scarlet
Second	Lemon & Mustard
Third	Ochre & Saffron
Fourth	Ocean & Royal
Fifth	Denim & Slate
Sixth	Amber & Honey
Seventh	Brick & Madder
Eighth	Cherry & Ruby
Ninth	Peacock & Teal

- (b) Who has achieved the same top-five position in the third and fourth races? [1]

Last year Team Red and Team Yellow tied with a total of 52 points each and, for the first time, the trophy was shared.

- (c) What was Team Blue's total last year? [1]

- (d) Explain why the result of the competition can never be a three-way tie. [1]

With one race left, the team totals are now:

Yellow	44 points
Red	41 points
Blue	31 points

- (e) Give the three possible final team totals for both Team Red and Team Yellow that would result in them sharing the trophy again today, after the final race. [3]

In the final race:

Team Blue's pairs are Denim & Teal, Ocean & Slate, and Peacock & Royal;

Team Yellow's pairs are Amber & Ochre, Honey & Lemon, and Mustard & Saffron.

- (f) Deduce Team Red's three pairs in the final race. [2]

There was also a tie in the individual competition last year, which resulted in two participants each receiving half of the cash prize.

The top five individuals after the fourth race today are:

Flame	29 points
Mustard	23 points
Scarlet	23 points
Saffron	21 points
Lemon	16 points

- (g) Explain why it is now certain that Flame or Scarlet or Mustard will win the whole of today's cash prize. [4]

- 3 In the political Assembly of Bolandia, there are 10 Representatives, one for each of the 10 constituencies in the country. Each Representative is elected by the residents of their constituency. Each Representative belongs to one (and only one) of three political parties.

In an election, the residents in a constituency may vote for one of three candidates (one from each of the three parties). The candidate who gets the most votes is chosen as Representative. It is not compulsory for any resident to vote.

For a constituency's election to be valid, both of the following conditions must be met:

- The winning candidate must have more votes than any other.
- At least 50% of the residents in the constituency must have cast a vote.

If either condition is not met then another election must be held in that constituency.

- (a) In a previous election, all 600 residents in a constituency voted, and the election was valid.

- (i) What is the minimum number of votes the winning candidate could have received? [1]
- (ii) What is the maximum number of votes that a losing candidate could have received? [1]

In this year's election, there were exactly 600 residents in each of the 10 constituencies.

- (b) If only 1 Representative from a particular party was elected in this year's election, what is the largest number of votes that party could have received across the whole country? [1]
- (c) If 6 Representatives from a particular party were elected in this year's election, what is the smallest number of votes that party could have received across the whole country? [2]
- (d) Suppose that, for one of the parties, the average number of votes it received for each of its elected Representatives was 150.

What is the largest number of residents who could have voted, for all parties, across the whole country? [2]

The following table gives the actual number of Representatives for the three parties who were elected in this year's election.

<i>Party</i>	<i>Representatives</i>
Green	6
Blue	3
Red	1

In spite of this result, it is possible that the Red party received more votes than the Green party in this year's election.

- (e) Find the greatest number of votes more than the Green party that the Red party could have received. [3]

- 4** Jane is responsible for ordering and installing 100 computers in an office for a new business that will open soon. The computers have been ordered from a company that assembles and delivers them. The company can assemble 5 computers each day.

Deliveries are sent at the end of the day in which all of the required computers have been assembled and arrive the following morning.

Jane is able to install 3 computers in one day.

To plan the installation of the computers, Jane numbers the working days as Day 1, Day 2, etc. On each of these days the company will be working on assembling Jane's computers, until all 100 have been assembled, and Jane will install computers if they have been delivered. On any day that a delivery arrives, it arrives before Jane starts work.

- (a)** If all the computers are sent in one delivery, on which Day will Jane finish installing them? [2]

Jane considers splitting her order into smaller deliveries so that she can install the computers from the first delivery while she is waiting for the second delivery to arrive.

- (b)** Suppose that Jane splits her order into two deliveries of 50 computers each.

On which Day will Jane finish installing all the computers? [1]

- (c) (i)** What is the earliest Day on which Jane could finish installing all the computers if the order is split into two deliveries? [2]

- (ii)** What is the smallest number of computers that could be in the first delivery, to have all the computers installed by the earliest Day? [1]

- (iii)** What is the largest number of computers that could be in the first delivery, to have all the computers installed by the earliest Day? [1]

Jane needs to have all of the computers installed before the office opens at the beginning of Day 38.

- (d) (i)** Explain why the latest day on which the first delivery could arrive is Day 4. [1]

- (ii)** How many deliveries would be needed to get all of the computers delivered and installed before the start of Day 38? Justify your answer. [3]

Jane decides that she should employ an assistant to help her to install the computers. The assistant will be able to install 2 computers each day, in addition to the 3 that Jane can install.

She wishes to employ the assistant for the smallest number of days possible so that only two deliveries would be required, but all the computers would still be installed on time. The first day that the assistant works will be the day that the second delivery arrives.

- (e)** If the assistant is employed for 10 days, what is the latest Day on which the first delivery could arrive? [2]

- (f)** What is the smallest number of days for which the assistant could be employed? Justify your answer. [2]

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